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CHAPTER

1 Basic Concepts and Properties

Theresa is conducting a chemistry experiment involving yellow phosphorus. The lab manual states that yellow phosphorus ignites at 34 degrees Celsius. What will the temperature be in degrees Fahrenheit? The formula for changing the temperature measured in degrees Celsius to degrees Fahrenheit is

$$F = 1.8\,C + 32$$

1–1 ■ Sets and real numbers

Set symbolism

We begin our study with a very simple, but important, mathematical concept—the idea of a **set.*** *A set is any collection of objects or things.* We want the sets that we deal with to be *well defined;* that is, given any object, we can determine whether the object is in a given set. For example, the set of old people is not well defined because the meaning of old is not clear. Whereas the set of people whose ages are greater than seventy years is a well-defined set. In mathematics, the idea of a set is used primarily to denote a group of numbers or the set of answers to a problem.

Any one of the things that make up a set is called a **member** or an **element** of that set. One way of writing a set is by listing the elements, separating them by commas, and including this listing within a pair of braces, $\{\ \}$. This way of representing a set is called the **listing** or **roster method.**

■ **Example 1–1 A**

1. Using set notation, write the days of the week that begin with the letter T.

{Tuesday, Thursday}

*Georg Cantor (1845–1918) is credited with the development of the ideas of set theory. He described a set as a grouping together of single objects into a whole.

2. Using set notation, write the digits that make up the telephone number for information, 555–1212.

   $\{5,1,2\}$

Note When we form a set, the elements within the set are never repeated and they can appear in any order.

▶ ***Quick check*** Using set notation, write the letters of the word "book." ■

We use capital letters A,B,C,D, and so on, to represent a set. When we wish to show that an element belongs to a particular set, we use the symbol $\in$, which is read "is an element of" or "is a member of." Consider the set $A = \{1,4,5\}$, which is read "the set A whose elements are 1, 4, and 5." If we want to say that 4 is an element of the set A, we write $4 \in A$.

A slash mark, /, is used in mathematics to negate a given symbol. Therefore if $\in$ means "is an element of," then $\notin$ means "is *not* an element of." To express the fact that 2 is not an element of the set A, we write $2 \notin A$.

■ **Example 1–1 B**

Using mathematical symbols, write the following statements.

1. 5 is an element of the set C. $5 \in C$
2. 4 is not an element of the set B. $4 \notin B$

▶ ***Quick check*** Write "7 is an element of the set A" using mathematical symbols. ■

Subsets

Suppose that P is the set of people in a class and W is the set of women in the same class. It is obvious that the members of W are also members of P. We say that W is a **subset** of P and use the symbol $\subseteq$ to indicate "is a subset of."

Definition of $A \subseteq B$

The set A is a subset of the set B, written $A \subseteq B$, if every element of A is also an element of B.

■ **Example 1–1 C**

Given the sets $A = \{1,2,3\}$, $B = \{1,2,3,4,5\}$, $C = \{2,3,5\}$, and $D = \{2,3,1\}$, determine if the following statements are true or false.

1. $A \subseteq B$. True, since all the elements in A are also in B.
2. $A \not\subseteq C$. True, since not all of the elements in A are in C. 1 is an element of A but is not an element of C.

Note $A \not\subseteq C$ is read "the set A is not a subset of the set C" and implies that there is at least one element in the set A that is not in the set C.

3. $C \subseteq D$. False, since not all of the elements in C are in D. 5 is an element of C but is not an element of D.
4. $A \subseteq D$. True, since all of the elements in A are in D. ■

In example 4, since the order of the elements within the set does not change the set, we can conclude that the sets A and D are the same set. Therefore we observe that *the definition of subsets allows a set to be a subset of itself.* That is, for any set A, $A \subseteq A$. We will now use the definition of subsets to define when two sets are equal.

Definition of $A = B$

The set A is said to be equal to the set B, written $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

Suppose that we wanted to form the set of the months of the year that begin with the letter X. Since there are no months that begin with the letter X, this set has no elements and is called the **empty set** or the **null set. A set that contains no elements is called the empty set or the null set and is denoted by the symbol $\emptyset$.**

By the definition of subsets, a set A is *not* a subset of a set B if there is at least one element in A that is not in B. The empty set, by definition, has no elements; therefore it can contain no elements that are not in another given set. We must then conclude that *the empty set is a subset of every set.*

Union and intersection of sets

If we wished to form the set of all the students who have blue eyes *or* blond hair, we would be combining the set of students with blue eyes with the set of students with blond hair. This new set that we have formed is the **union** of the two original sets.

Definition of $A \cup B$

The union of the sets A and B, written $A \cup B$, is the set of all elements that are in A or in B or in both A and B.

Example 1–1 D

Given the sets $A = \{2,3,5\}$, $B = \{1,3,7\}$, and $C = \{2,4,6\}$, form the following sets.

1. $A \cup B$

A union B consists of those elements that appear in A, $\{2,3,5\}$, or those in B, $\{1,3,7\}$, or those in both A and B, $\{3\}$. Therefore

$$A \cup B = \{1,2,3,5,7\}$$

2. $B \cup C = \{1,3,7\} \cup \{2,4,6\} = \{1,2,3,4,6,7\}$ ■

If we wished to form the set of all the students who have blue eyes *and* blond hair, we would be forming a new set of students that possess the characteristic of having *both* blue eyes and blond hair. This new set that we have formed is the **intersection** of the two original sets.

Definition of $A \cap B$

The intersection of the sets A and B, written $A \cap B$, is the set of only those elements that are in both A and B.

Example 1–1 E

Given the sets $A = \{1,2,3\}$, $B = \{2,3,4,5\}$, $C = \{4,5\}$, and $D = \{2,3,6,7\}$, form the following sets.

1. $A \cap B$

$A \cap B = \{1,2,3\} \cap \{2,3,4,5\} = \{2,3\}$

A intersection *B* consists of only those elements that appear in *both A* and *B*

2. $A \cap C = \{1,2,3\} \cap \{4,5\}$

There are no elements common to both *A* and *C*

$A \cap C = \emptyset$

The intersection is the empty set

3. $A \cup (B \cap C)$

$= \{1,2,3\} \cup (\{2,3,4,5\} \cap \{4,5\})$

Operations enclosed within a grouping symbol must be performed first

$= \{1,2,3\} \cup \{4,5\}$

Perform the intersection of *B* and *C*

$= \{1,2,3,4,5\}$

Perform the union of $B \cap C$ with *A* ■

When the intersection of two sets is the empty set, as in example 2, we say that the two sets are **disjoint.** That is, *the sets* A *and* C *are disjoint if and only if* $A \cap C = \emptyset$.

The set of real numbers

Our study of algebra will be primarily concerned with the set of **real numbers** and its properties. We shall now review the set of real numbers and its major subsets.

In each of the previous examples, we could determine the exact number of elements in a set. This type of set is called a **finite** set. Each of the major sets of numbers that make up the set of real numbers has an unlimited number of elements. This is called an **infinite** set.

The set of real numbers is made up of the following sets of numbers.

I. The set of **natural numbers,** or counting numbers, is defined by

$\{1,2,3,4, \cdots\}$

and denoted by N.

Note The three dots tell us to continue this pattern indefinitely. That is, there is no last natural number.

II. The set of **whole numbers** is defined by

$\{0,1,2,3,4, \cdots\}$

and denoted by W.

III. The set of **integers** is defined by

$\{\cdots, -2, -1, 0, 1, 2, \cdots\}$

and denoted by J.

To express certain sets of numbers, we need to define the mathematical concept of a **variable. A variable is a symbol (generally a lowercase letter) that acts as a placeholder for an unspecified number.** When we use a variable to represent a set of numbers, the set of numbers is called its **replacement set** or **domain.** For example,

$$\{x \mid x \text{ is a natural number less than } 6\}$$

is read "the set of all elements x such that x is a natural number less than 6." This notation defines the set $\{1,2,3,4,5\}$. The bar is read "such that" and the notation is called **set-builder notation.** In general, set-builder notation has the pattern shown in figure 1–1.

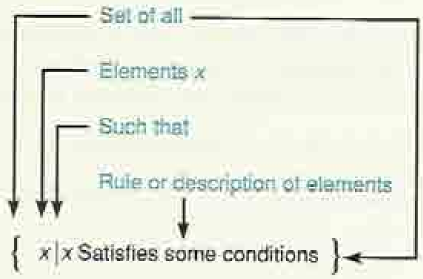


Figure 1–1

Example 1–1 F

List the elements of the following sets.

1. $\{x \mid x \text{ is an integer between } -4 \text{ and } 2\}$
   $= \{-3, -2, -1, 0, 1\}$

   The word "between" indicates not to include -4 and 2

2. $\{x \mid x \text{ is a whole number greater than } 10\}$
   $= \{11, 12, 13, 14, \cdots\}$

   The words "greater than" indicate that we start with the first whole number after 10

Note Since this is an infinite set, we set up a pattern for the numbers and place three dots after the last number to indicate that this pattern continues indefinitely.

3. $\{x \mid x \text{ is a natural number less than } 1\}$

   Since there are no natural numbers less than 1, the set is empty, $\emptyset$.

IV. The set of **rational numbers** is defined by

$$\left\{\frac{p}{q}\,\middle|\, p \text{ and } q \in J,\ q \neq 0\right\}$$

and denoted by Q. The set-builder notation symbolizes that the set of rational numbers, Q, will consist of all the numbers that can be represented by the quotient of two integers where the denominator is not zero.

A second way that the set of rational numbers can be defined is

$\{x|$ the decimal representation of x is either terminating or repeating$\}$

Examples of terminating and repeating decimals are

$$\frac{1}{2} = 0.5,\quad \frac{1}{3} = 0.\overline{3},\quad -\frac{1}{6} = -0.1\overline{6},\quad -\frac{5}{4} = -1.25$$

where a bar placed over a number or group of numbers indicates that the number(s) repeat indefinitely.

V. The set of **irrational numbers** is defined by

$\{x|$ the decimal representation of x is nonterminating and nonrepeating$\}$

and denoted by H. Examples of irrational numbers are

$$\sqrt{3},\quad -\sqrt{5},\quad \pi,\quad \frac{\sqrt{2}}{2}$$

The decimal representation of an irrational number will never terminate. We cannot find a repeating pattern of digits, no matter how many digits we write past the decimal point.

Note By using a calculator, the previous numbers can be represented by the following approximations to three decimal places:

$$\sqrt{3} \approx 1.732,\quad -\sqrt{5} \approx -2.236,\quad \pi \approx 3.142,\quad \frac{\sqrt{2}}{2} \approx 0.707$$

The symbol $\approx$ is read "is approximately equal to." π is the distance around a circle (circumference) divided by the distance across the circle through the center (diameter). Common approximations for π are 3.14 and $\frac{22}{7}$.

VI. The set of **real numbers** is defined by

$\{x|x \in Q \text{ or } x \in H\}$

and denoted by R.

Note Whenever we encounter a problem and a specific replacement set or domain for the variable is not indicated, it will be understood that we are dealing with the set of real numbers.

All of the sets that we have considered thus far are subsets of the set of real numbers. Figure 1–2 shows this relationship.

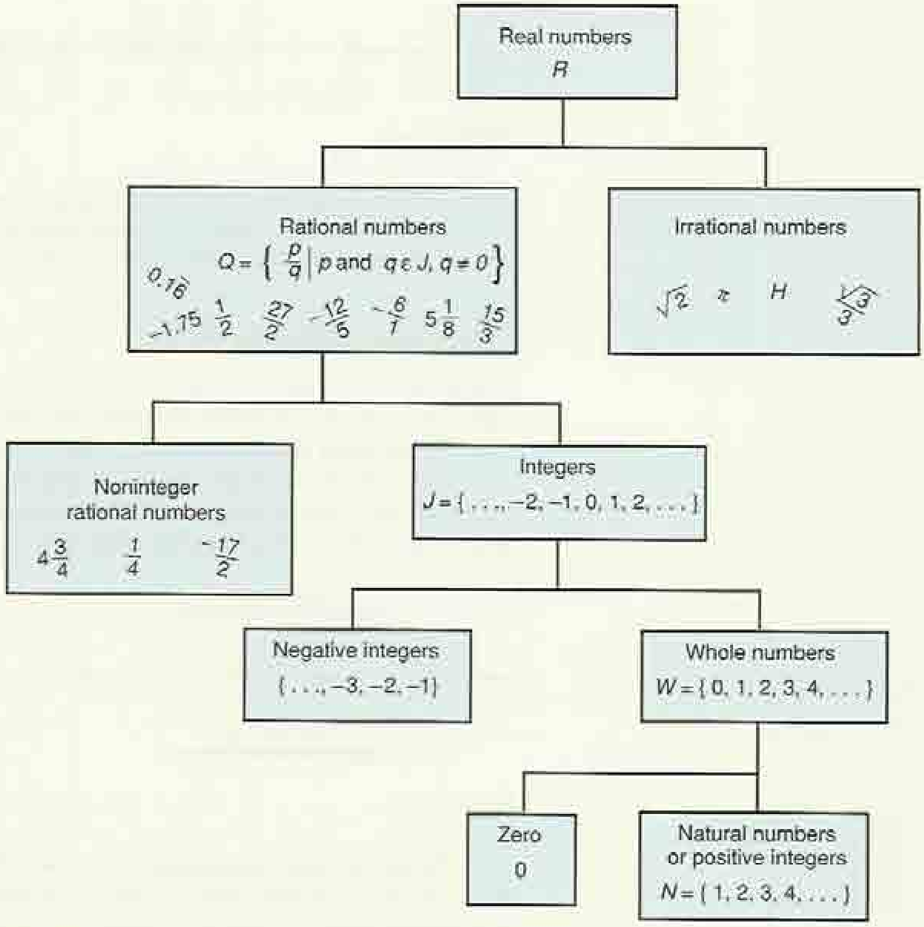


Figure 1–2

The real number line

To visualize the set of real numbers, we use a figure called a **number line.** Any real number can be located on the number line (see figure 1–3).

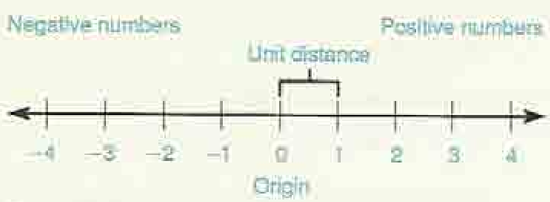


Figure 1–3

The number associated with each point on the number line is called the **coordinate** of the point. The point associated with each number is called the **graph** of that number. In figure 1–4, the numbers -2, $-\frac{1}{3}$, $\frac{3}{4}$, $\sqrt{3}$, and 3 are the coordinates of the points indicated on the line by solid circles. These solid circles are the graphs of the numbers -2, $-\frac{1}{2}$, $\frac{3}{4}$, $\sqrt{3}$, and 3.

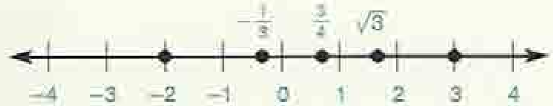


Figure 1–4

Note The coordinate $\sqrt{3}$ represents an irrational number. To graph this point, we can use a decimal approximation from a calculator, $\sqrt{3} \approx 1.732$. The word "number" used in this book means "a real number."

Order and absolute value

The direction we move on the number line is also important. If we move to the right, we move in a positive direction and our numbers **increase.** If we move to the left, we move in a negative direction and our numbers **decrease.**

If we choose arbitrary points on the number line and represent them by a and b, where a and b represent some *unspecified* numbers, we observe that there is an **order** relationship between a and b as in figure 1–5.

Figure 1–5

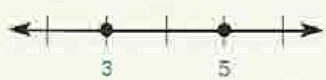
When the point associated with a is to the left of the point associated with b, we say that a is **less than** b, which in symbols is $a < b$. We might also say that b is **greater than** a, which in symbols is $b > a$. The symbols $<$ (is less than) and $>$ (is greater than) are inequality symbols called **strict inequalities** and denote an **order relationship** between numbers.

Example 1–1 G

Graph the following pairs of numbers and insert the correct inequality symbol between the numbers to get a true statement.

1. 3 and 5

The graph would be



Since 3 lies to the left of 5, we say that 3 is less than 5, or symbolically $3 < 5$.

Note $3 < 5$, read "3 is less than 5," can also be stated as $5 > 3$, read "5 is greater than 3." No matter which inequality symbol we use, the inequality symbol always points to the *lesser* number.

2. -90 and -100
The graph would be

Since -90 lies to the right of -100, we say that -90 is greater than -100, or symbolically $-90 > -100$. ■

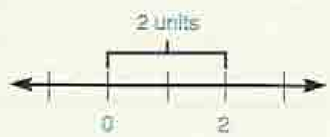
There are two other inequality symbols, called **weak inequalities,** which are **is less than or equal to,** $\leq$, and **is greater than or equal to,** $\geq$. The weak inequality symbol $\leq$, is less than or equal to, combines the relationship of is less than ($<$) with the relationship of equality ($=$). The weak inequality symbol $\geq$, is greater than or equal to, combines the relationship of is greater than ($>$) with the relationship of equality ($=$).

As we study the number line, we see a very useful property called **symmetry.** The numbers are symmetrical with respect to the origin. That is, if we go two units to the right of 0, we come to the number 2, and if we go two units to the left of 0, we come to the *opposite* of 2, which is -2. This idea of how far a given number is from the origin is called the **absolute value** of that number. *The absolute value of a number is the undirected distance that the number is from the origin.* The symbol for absolute value is $| \quad |$

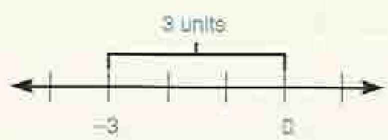
■ Example 1–1 H

Find the indicated absolute values.

1. $|2| = 2$



2. $|-3| = 3$



■

Note The absolute value of a number is *never* negative. That is, for every $x \in R, |x| \geq 0$.

When we wish to determine the absolute value of a positive number or zero, $x \geq 0$, the absolute value of the number is simply the original number, x. When we wish to determine the absolute value of a negative number, $x < 0$, we find that the absolute value of a negative number is the opposite of the original number. We symbolically write this as $-x$, read "the opposite of x." We can now state the definition of absolute value symbolically.

Definition of $|x|$

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

Since x represents a negative number ($x < 0$), the absolute value of x is the opposite of x (or $-x$) and will be a positive number

■ **Example 1–1 I**

Find the indicated absolute values.

1. $|4| = 4$ **2.** $|-9| = -(-9) = 9$ **3.** $-|-7| = -(7) = -7$

Note The negative sign in front of the absolute value in example 3 indicates that we want the opposite of the absolute value. Therefore, the answer is -7. ■

Mastery points

Can you

- List the elements of a set?
- Use set symbolism?
- Determine when a set is a subset of another set?
- Determine when sets are equal?
- Perform the operations of union and intersection on sets?
- Use set-builder notation?
- Graph a number?
- Determine the coordinate of a point?
- Determine which of two real numbers is greater?
- Find the absolute value of a number?
- Use mathematical symbols?

Exercise 1–1

Write each set by listing the elements. See example 1–1 A.

Example Using set notation, write the letters of the word "book."

Solution $\{b,o,k\}$ The letters are placed within braces in any order, separated by commas, and we do not repeat any of the elements.

1. The days of the week that begin with the letter S

2. The months of the year that begin with the letter A

3. The even integers between 9 and 15

4. The months of the year with less than 28 days

5. The months of the year that begin with the letter C

6. The days of the week that begin with the letter A

Use mathematical symbols to write the following statements. See examples 1–1 B and C.

Example 7 is an element of the set A.

Solution $7 \in A$ $\in$ is the symbol for "is an element of"

7. A is a subset of D.

8. B is not a subset of C.

9. The null set is a subset of B.

10. The null set is not an element of C.

11. The set whose elements are 6, 7, 8

12. The set whose elements are 1, 4, 6

Given the sets $A = \{2,4,6\}$, $B = \{1,3,5\}$, $C = \{1,2,3,4,5,6\}$, $D = \{3,1,5\}$, and $E = \{1,2,3,4,5,6,7,8\}$, determine if the following statements are true or false. See example 1–1 C.

13. $2 \in A$ **14.** $1 \in B$ **15.** $A \subseteq C$ **16.** $B \subseteq E$ **17.** $A \not\subseteq E$

18. $C \subseteq E$ **19.** $B = D$ **20.** $A = D$ **21.** $\emptyset \subseteq A$ **22.** $A \subseteq D$

Given the sets $A = \{1,2,3,4\}$, $B = \{2,3,4,5,6\}$, $C = \{2,3,4\}$, and $D = \{7,8,9\}$, form the following sets. See examples 1–1 D and E.

23. $A \cup C$ **24.** $B \cup C$ **25.** $A \cup D$ **26.** $B \cup D$ **27.** $A \cap D$

28. $C \cap D$ **29.** $(C \cup B) \cap A$ **30.** $(D \cap C) \cup B$ **31.** $A \cup (C \cap D)$ **32.** $(B \cap C) \cup A$

Graph the following sets of numbers on a number line. See example 1–1 G.

33. $\{-3,-1,2,4,5\}$ **34.** $\{-4,-3,-2,1,3\}$ **35.** $\left\{-\frac{7}{4},0,\frac{5}{4},3,5\right\}$

36. $\{-\sqrt{3},0,\sqrt{2},3,6\}$ **37.** $\{-5,-\sqrt{5},\sqrt{3},3,5\}$ **38.** $\{-4,-2,0,\sqrt{5},\sqrt{7}\}$

Write the value of the following numbers. See examples 1–1 H and I.

39. $|-3|$ **40.** $|-5|$ **41.** $|0|$ **42.** $\left|\frac{1}{4}\right|$ **43.** $-|-2|$

44. $-|-7|$ **45.** $-|4|$ **46.** $-|5|$

Replace the comma with the proper strict inequality symbol, $<$ or $>$, between the following numbers to get a true statement. See example 1–1 G.

47. 4, 9 **48.** $-4, -8$ **49.** $-3, -11$ **50.** $-12, -5$ **51.** $-15, -10$

52. $0, -5$ **53.** $-4, 0$

Use mathematical symbols to write each of the following statements.

Example 0 is not an element of the set of natural numbers.

Solution $0 \notin N$ The / mark through the $\in$ symbol means "is **not** an element of the set"

54. The set of whole numbers is not a subset of the set of natural numbers.

55. The set of irrational numbers intersected with the set of rational numbers is the null set.

56. The union of the set of rational numbers with the set of irrational numbers is the set of real numbers.

57. $\{8\}$ is a subset of the set of natural numbers.

58. 8 is not a subset of the set of natural numbers.

59. $\{8\}$ is not an element of the set of natural numbers.

60. 8 is an element of the set of natural numbers.

Determine from the given information whether the answer to each of the following statements is yes or no. The sets A, B, and C are general sets and are not the same as the sets in the previous exercises.

Example If $A \subseteq B$ and $2 \in A$, must 2 be an element of B?

Solution Yes. From the definition of subsets, every element in A must be in B. Therefore if $2 \in A$ and $A \subseteq B$, then $2 \in B$.

61. If $A \subseteq B$ and $3 \in A$, must 3 be an element of B?

62. If $A \subseteq B$ and $7 \in B$, must 7 be an element of A?

63. If $A \not\subseteq B$ and $6 \in A$, does this mean that $6 \notin B$?

64. If $A \not\subseteq B$ and $1 \notin A$, does this mean that 1 must be an element of B?

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65. If $A \subseteq B$ and $B \subseteq C$ and $4 \in A$, must 4 be an element of C?

66. If $A \subseteq B$ and $B \subseteq A$, does this mean that $A = B$?

67. If $A \subseteq B$ and $B \subseteq C$, must A be a subset of C?

68. If $A \subseteq B$ and $B \subseteq C$ and $5 \in A$, must 5 be an element of C?

69. If $A \subseteq B$ and $B \subseteq A$ and $8 \notin A$, must 8 be an element of B?

70. If $A \not\subseteq B$, can $B \subseteq A$?

71. Is $\emptyset \subseteq \emptyset$?

72. Is $\emptyset \subseteq B$?

73. Does $\emptyset \cap A = \emptyset$?

74. Does $\emptyset \cup A = A$?

75. Does $A \cap A = A$?

76. Does $A \cup A = A$?

1–2 ■ Operations with real numbers

Addition

In this section, we will review the rules for addition and subtraction of real numbers. We use the minus sign $(-)$ to indicate a negative number and the plus sign $(+)$ to indicate a positive number. A **real number** consists of two parts: its **absolute value** and its **sign** (see figure 1–6).

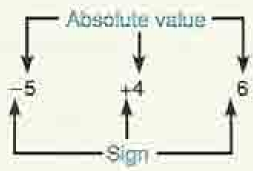

Figure 1–6

We can summarize the rules for adding real numbers.

Addition of real numbers

1. If the signs are the same, we add their absolute values. The answer is given their common sign.
2. If the signs are different, we subtract the lesser absolute value from the greater absolute value. The answer is given the sign of the number with the greater absolute value.

■ Example 1–2 A

Add the following.

1. $(+3) + (+6) = (+9)$
   Absolute value
   Common sign

2. $(-4) + (-9) = (-13)$
   Absolute value
   Common sign

3. $(-8) + (+5) = (-3)$
   Absolute value
   Sign of the number with the greater absolute value

4. $(-6) + (+10) = (+4)$

Absolute value

Sign of the number with the greater absolute value

Subtraction

We are already familiar with the operation of subtraction in problems such as $12 - 8 = 4$. If we were to add (-8) instead of subtracting $(+8)$, we observe that the results are the same, that is

Opposite of +8

$$(+12) - (+8) = (+12) + (-8) = (+4)$$

Change to addition

This leads us to the following definition of subtraction.

Definition of subtraction

For any two real numbers a and b, the difference of a and b is

$$a - b = a + (-b)$$

Concept

a minus b is equal to a plus the opposite of b.

Our steps to carry out the subtraction are as follows:

Subtraction of real numbers

Step 1 We change the operation from subtraction to addition.

Step 2 We change the sign of the number that follows the subtraction symbol.

Step 3 We perform the addition using our rules for adding real numbers.

Example 1–2 B

Subtract the following.

		Stays as it was	*Change to addition*	*Changes to the opposite*		*Add using the rules for addition*
1. $(+7) - (+4)$	=	$(+7)$	+	(-4)	=	3
2. $(-11) - (+3)$	=	(-11)	+	(-3)	=	-14
3. $(-14) - (-10)$	=	(-14)	+	$(+10)$	=	-4
4. $(-2) - (-12)$	=	(-2)	+	$(+12)$	=	10

▶ ***Quick check*** $(-6) - (-14)$

When a series of numbers involving addition and subtraction are written horizontally, we can mentally change the operation of subtraction to addition and perform the operations in order from left to right as they occur. For example, consider

$$10 - (-2) + 7 + (-4) - 8$$

Changing the indicated subtractions to additions, we obtain the indicated sum.

$$10 + (+2) + 7 + (-4) + (-8) = 7$$

Many times, part of a problem will have a group of numbers enclosed within grouping symbols such as parentheses ( ), brackets [ ], or braces { }. If a quantity is enclosed within a grouping symbol, we treat the quantity within as a single number. Thus given

$$14 - (5 - 7) + (4 + 8) - (6 - 2)$$

we perform the operations within the parentheses first to get

$$14 - (-2) + (12) - (4)$$

and changing the subtractions to additions

$$14 + (+2) + (12) + (-4)$$

and adding from left to right

$$\begin{aligned} &= 16 + (12) + (-4) \\ &= 28 + (-4) \\ &= 24 \end{aligned}$$

Example 1–2 C

Perform the indicated operations.

1. $(-14) - (-8) + (-7) - (4)$
Change the subtractions to additions.

$$= (-14) + (+8) + (-7) + (-4)$$

Perform the addition from left to right.

$$\begin{aligned} &= (-6) + (-7) + (-4) \\ &= (-13) + (-4) \\ &= -17 \end{aligned}$$

2. $12 - (4 - 7) + (8 - 3) + (5 - 9)$

$$\begin{aligned} &= 12 - (-3) + (5) + (-4) && \text{Simplify within parentheses} \\ &= 12 + (+3) + (5) + (-4) && \text{Change to addition} \\ &= 15 + (5) + (-4) && \text{Add from left to right} \\ &= 20 + (-4) \\ &= 16 \end{aligned}$$

Multiplication

We are already familiar with the fact that the product of two positive numbers is positive. We can see this by considering multiplication to be repeated addition. For example, $3 \cdot 4$ means the sum of the three 4s, that is,

$$3 \cdot 4 = 4 + 4 + 4 = 12$$

The number 12 is called the **product** of 3 and 4, furthermore 3 and 4 are called **factors** of 12. *The numbers or variables in an indicated multiplication are referred to as the factors of the product.*

We can summarize the rules of multiplication of real numbers.

Multiplication of real numbers

1. When we multiply two numbers having the same sign, the product will be positive.
2. When we multiply two numbers having different signs, the product will be negative.

Example 1–2 D

Multiply the following.

1. $(+4)(+8) = +32$
2. $(-6)(-3) = +18$
3. $(-4)(+5) = -20$
4. $(+2)(-8) = -16$

▶ ***Quick check*** $(-5)(-7)$

Exponential notation

Consider the indicated products

$$4 \cdot 4 \cdot 4 = 64$$

and

$$3 \cdot 3 \cdot 3 \cdot 3 = 81$$

A more convenient way of writing $4 \cdot 4 \cdot 4$ is 4^3, which is read "4 to the third power" or "4 cubed." We call the number 4 the **base** of the expression and the number 3 to the upper right the **exponent.**

Thus

Base Exponent

Expanded form $\longrightarrow 4 \cdot 4 \cdot 4 = 4^3 = 64 \longleftarrow$ Standard form

Exponential form

In like fashion,

$$3 \cdot 3 \cdot 3 \cdot 3$$

may be written 3^4, where 3 is the base and 4 is the exponent, and the expression is read "3 to the fourth power." Then

$$3 \cdot 3 \cdot 3 \cdot 3 = 3^4 = 81$$

Notice that *the exponent tells how many times the base is used as a factor in an indicated product.* We call this form of a product the **exponential form.** That is, the exponential form of the product $3 \cdot 3 \cdot 3 \cdot 3$ is 3^4.

We will now relate the idea of exponents to signed numbers. Consider the following examples.

Example 1–2 E

Perform the indicated multiplication.

1. $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$ — The exponent, 4, indicates how many times the base, 2, is used as a factor
2. $(-2)^4 = (-2)(-2)(-2)(-2) = +16$ — Even number of negative factors
3. $(-2)^3 = (-2)(-2)(-2) = -8$ — Odd number of negative factors

Remember that when we have a negative number, we place it inside parentheses. With this idea in mind, we can see that there is a definite difference between $(-2)^4$ and -2^4. In the first case, the parentheses denote that this is a negative number to a power, $(-2)^4 = (-2)(-2)(-2)(-2) = +16$. In the second case, since there are no parentheses around the number, we understand that this is *not* (-2) to a power, but rather the opposite of what we get for an answer when we find 2^4 as follows:

$$-2^4 = -(2^4) = -(2 \cdot 2 \cdot 2 \cdot 2) = -(16) = -16$$

Example 1–2 F

Perform the indicated multiplication.

1. $(-3)^3 = (-3)(-3)(-3) = -27$
2. $-3^3 = -(3 \cdot 3 \cdot 3) = -27$ This is the opposite of 3^3, $-(3^3)$
3. $(-3)^4 = (-3)(-3)(-3)(-3) = +81$
4. $-3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$ This is the opposite of 3^4, $-(3^4)$

Division

We studied subtraction of signed numbers by defining subtraction in terms of addition. We shall use a similar approach for division of signed numbers by defining division in terms of multiplication.

Definition of division

The quotient of any two real numbers a and b ($b \neq 0$) is the unique real number q such that $a = bq$. In symbols,

$$\frac{a}{b} = q \text{ if and only if } a = bq$$

Note In the definition, a is called the dividend, b is the divisor, and q is the quotient.

We can summarize our rules for multiplication and division of real numbers.

Multiplication and division of real numbers

1. When we multiply or divide two numbers having the same sign, our answer will be positive.
2. When we multiply or divide two numbers having different signs, our answer will be negative.

Example 1–2 G

Perform the indicated division.

1. $\dfrac{(-14)}{(-7)} = +2$
2. $\dfrac{(-24)}{(+8)} = -3$
3. $\dfrac{(+36)}{(-9)} = -4$
4. $\dfrac{(-2)(-8)}{(-4)} = \dfrac{(+16)}{(-4)} = -4$
5. $\dfrac{(+6)(-3)}{(-9)} = \dfrac{(-18)}{(-9)} = +2$

Division involving zero

In section 1–1, we defined a rational number to be any real number that can be expressed as a quotient of two integers where the divisor is not zero. The number zero, 0, is the only number that we cannot use as a divisor. To see why we exclude zero as a divisor, recall that we check a division problem by multiplying the divisor times the quotient to get the dividend. If we apply this idea in connection with zero as a divisor, we observe the following. Suppose there were a number q, such that $3 \div 0 = q$. Then $q \cdot 0$ would have to be equal to 3 for our answer to check, but this product is zero regardless of the value of q. Therefore, we cannot find an answer for this problem and we say that the answer is *undefined*. If we try to divide zero by zero and again call our answer q, we have $0 \div 0 = q$, and when we check our work, $0 \cdot q = 0$, we see that any value for q will work. In this situation, we say our answer is *indeterminate*. We therefore decide that **division by zero is not allowed.**

It is important to note that, although division by zero is not allowed, this does not extend to the division of zero by some other number. We can see that $0 \div (-4) = 0$ since $(-4) \cdot 0 = 0$. Thus *the quotient of zero divided by any number other than zero is always 0.*

Division involving zero

If a is any real number except 0,	example
$\frac{0}{a} = 0$	$\frac{0}{4} = 0$
$\frac{a}{0}$ is undefined	$\frac{4}{0}$ is undefined

$\frac{0}{0}$ is indeterminate

Problem solving

To solve the following word problems, we will represent gains by positive real numbers and losses by negative real numbers.

Example 1–2 H

Choose a variable to represent the unknown quantity and find its value.

1. A board that is 8 feet long is joined with a board that is 5 feet long. What is the total length of the board?

   Let x equal the total length of the board. To find the total length we must *add* the individual lengths. Thus

   $x = 8 + 5 = 13$

   The total length of the board is 13 feet.

2. On a given winter's day in Chicago, the temperature was 14° in the afternoon. By 9 P.M. the temperature was −4°. How many degrees did the temperature fall from afternoon to 9 P.M.?

   Let x = the number of degrees fall in temperature. We must find the difference between 14° and −4°. Thus

   $$x = 14 - (-4) = 14 + 4 = 18$$

   There was an 18° fall in temperature.

3. If \$13.58 is spent on 14 audio tapes, how much did each tape cost?

   Let c = the cost of each tape. We must divide \$13.58 by 14. Thus

   $$c = \frac{13.58}{14} = 0.97$$

   Each audio tape costs 97¢. ■

Mastery points

Can you

- Add real numbers?
- Subtract real numbers?
- Perform addition and subtraction in order from left to right?
- Treat a quantity within a grouping symbol as a single number?
- Multiply real numbers?
- Divide real numbers?
- Use exponents?
- Apply the rules of division involving zero?
- Solve word problems?

Exercise 1–2

Find each sum or difference. See examples 1–2 A, B, and C.

Example $(-6) - (-14)$

Solution $= (-6) + (+14)$ Change the subtraction to addition, change −14 to +14

$= 8$ Add using the rules for addition

1. $(+6) - (+4)$
2. $(+7) - (-5)$
3. $(-8) - (+6)$
4. $(+7) - (+12)$
5. $(+10) + (-6) + (-3)$
6. $(-5) - (-1)$
7. $(+10) - (-8)$
8. $(-12) + 0$
9. $(-7) - 0$
10. $(+16) + (-7)$
11. $(+9) + (-13)$
12. $0 + (-4)$
13. $(-20) - 0$
14. $6 - 9 + 11 - 8$
15. $-9 - 8 - 7 + 12$
16. $-10 - 12 + 18 + 4$
17. $9 - 4 + 6 - 5 + 7$
18. $(8 - 3) - 6 + (7 + 5)$
19. $-6 - 5 + 8 - (12 - 6)$
20. $14 - [(-4) - (-6)] + (7 - 9)$
21. $[(-6) - 5] + 4 - (16 - 6)$
22. $[4 - (-8)] - 6 - (2 - 11)$

Perform the indicated operations if possible. See examples 1–2 D, E, F, G, and H.

Example $(-5)(-7)$

Solution $= 35$ Signs are the same: positive product

23. $(-2)(-5)$
24. $(-6)(-8)$
25. $(+4)(-7)$
26. $(+5)(-6)$
27. $(-9)(+3)$
28. $(-11)(+4)$
29. $(-3)(-1)(-8)$
30. $(-7)(-2)(-6)$
31. $(+4)(-2)(-6)$
32. $(+5)(-3)(+2)$
33. $(-6)(+10)(+4)$
34. $(+7)(-3)(-2)$
35. $(-4)(-5)(-1)(-3)$
36. $(-9)(+2)(0)(-4)$
37. $(-8)(-10)(+6)(0)$
38. -5^2
39. -2^6
40. $(-4)^2$
41. $(-6)^2$
42. $(-3)^3$
43. $(-4)^3$
44. -5^3
45. -2^3
46. -6^3
47. $\dfrac{(-20)}{(-10)}$
48. $\dfrac{(-32)}{(-8)}$
49. $\dfrac{(+32)}{(-8)}$
50. $\dfrac{(-22)}{(-11)}$
51. $\dfrac{(-21)}{(-7)}$
52. $\dfrac{(-34)}{(-17)}$
53. $\dfrac{(-27)}{(+3)}$
54. $\dfrac{0}{(-8)}$
55. $\dfrac{0}{0}$
56. $\dfrac{(-4)(-3)}{(-6)}$
57. $\dfrac{(-20)(+2)}{(-4)}$
58. $\dfrac{(+18)(+2)}{(-6)}$
59. $\dfrac{(-6)(0)}{(-2)}$
60. $\dfrac{(-8)(0)}{(-2)(+4)}$
61. $\dfrac{(-18)(+12)}{(-9)(+6)}$
62. $\dfrac{(-8)(-6)}{(-2)(0)}$
63. $\dfrac{(-4)(-3)}{(0)(-2)}$
64. $\dfrac{(-6)^2}{(-2)(-2)}$
65. $\dfrac{-10^2}{(-5)(-5)}$
66. $\dfrac{(-12)(0)}{(0)(-3)}$

67. Harry owes Kay and Bill \$153 and \$56, respectively, and Donna owes Harry \$121. In terms of positive and negative symbols, how does Harry stand monetarily?

68. If Helen has \$46 just after paying off a debt of \$37, how much money did she have before paying off the debt?

69. The temperatures on January 15 in Chicago for the last six years are 14°, −6°, −4°, 19°, 7°, and −12°. What is the average temperature on January 15 for the last six years? (*Hint:* The average is found by adding all of the values and then dividing that sum by the number of values.)

70. The temperatures on February 1 in Ogema, Wisconsin, for the last eight years are −20°, −11°, 0°, −6°, −9°, −11°, 4°, and −3°. What is the average temperature on February 1 for the last eight years? (Refer to exercise 69.)

71. Eric Dickerson carried the ball six times as follows: 14-yard gain, 4-yard gain, 6-yard gain, 8-yard loss, 10-yard gain, and 4-yard gain. Represent the gains as positive integers and the losses as negative integers and find his average for the six carries. (Refer to exercise 69.)

72. Over a six-day period, the price of a particular stock suffered losses of \$5 the first two days and \$4 the last four days. If the stock originally sold for \$88, what was its price after the six-day period?

73. Jeff acquires a debt of \$10 each day for seven days. If we represent a \$10 debt by (−10), write a statement of the change in his assets after seven days. What is the change?

74. Eight days ago Joyce had \$48 more in assets than she has today. Let eight days ago be represented by (−8). Write a statement for her daily change in assets if the change is the same each day. What is this daily change?

75. The temperature on a given day in Anchorage, Alaska, was −10° F. The temperature went down 14° F. What was the final temperature that day?

76. A TWA flight is flying at an altitude of 27,000 feet. It suddenly hits an air pocket and drops 1,800 feet. What is its new altitude?

77. Mary Ann has \$125 in her checking account. She deposits \$28, \$14, and \$17. She then writes checks for \$52 and \$64. What is her final balance in the checking account?

78. An auditorium contains 68 rows of seats. If each row contains 40 seats, how many people can be seated in the auditorium?

79. In a classroom, there are 4 rows of desks. If each row contains 8 desks, how many students will the classroom hold?

80. Tom received money on his birthday from four different people. He received \$10, \$6, \$8, and \$5. How much money did Tom receive for his birthday?

81. Nancy sold 12 glasses of lemonade at her corner stand. If she charged 20 cents a glass, how much did she make in sales?

82. Lisa was born in 1963. How old will she be in the year 2000?

83. A chemist now has 125 ml (milliliters) of acid and she needs 415 ml of the acid. How much more is needed?

84. Beth has \$175 and wants to buy a television for \$485. How much will she owe?

85. The top of Mt. Everest is 29,028 feet above sea level and the top of Mt. McKinley is 20,320 feet above sea level. How much higher is the top of Mt. Everest than the top of Mt. McKinley?

86. Death Valley is 282 feet below sea level (-282). What is the difference in the altitude between Mt. McKinley and Death Valley? (See exercise 85.)

87. Mrs. Spencer paid \$25.20 for 14 watermelons for her fruit market. How much did each watermelon cost her?

88. A carpenter wishes to cut a 12-foot board into 4 pieces that are all the same length. Find the length of each piece.

89. A man drove 374 miles and used 11 gallons of gasoline. How many miles did he drive on each gallon of gasoline?

90. A grocer averages selling 68 gallons of milk each day. How many gallons of milk does he sell in two weeks? (Assume the grocer is open seven days per week.)

91. Rob's blood pressure was 127 over 82. If the first number rose by 6 and the second number dropped by 4, what is the new reading?

92. If Martha drove 252 miles in six hours, how many miles did she travel each hour (in miles per hour) if she drove at a constant speed?

93. Alice can work 62 math problems per hour. How many hours would it take her to work 1,674 problems?

94. Al can type 2,436 words in 29 minutes. How many words can he type per minute?

95. The barometric pressure rose 8 mb (millibars) and then dropped 12 mb. Later that day the pressure dropped another 4 mb and then rose 10 mb. What was the gain or loss in barometric pressure that day?

1–3 ■ Properties of real numbers

In mathematics, we begin the development of a set of properties of numbers by making certain assumptions. These assumptions are called **axioms** and are formal statements about numbers that we assume to be true. Such assumptions may arise from the observation of a number of instances of a specific situation or may just be statements that we assume are always valid. This appears to give us freedom to introduce as many such assumptions as we wish, but it is desirable that all such axioms lead to useful consequences. The fewer axioms that we use, the more powerful will be our system.

The first of our assumptions has to do with equality. An **equality** is a mathematical statement that two symbols, or groups of symbols, are names for the same number. For example, $2 + 3$ and $4 + 1$ are different symbols for the number 5, and we can state this by the equality

$$2 + 3 = 4 + 1$$

We shall assume that real numbers have the following properties of equality.

Equality properties of real numbers

For all real numbers a, b, and c

Reflexive property of equality
$a = a$

Symmetric property of equality
If $a = b$, then $b = a$

Transitive property of equality
If $a = b$ and $b = c$, then $a = c$

Substitution property of equality
If $a = b$, then a may be replaced by b or b may be replaced by a in any statement without changing the truth or falsity of the statement.

■ Example 1–3 A

The following statements are examples of the properties of equality.

1. $10 = 10$ — Reflexive property
2. If $3 = x$, then $x = 3$ — Symmetric property
3. If $m = n$ and $n = 5$, then $m = 5$ — Transitive property
4. If $x = 2$ and $x + 1 = 3$, then $2 + 1 = 3$ — Substitution property ■

In section 1–1, we discussed inequalities and the idea of order on the real number line. We will now state two properties of inequality of real numbers.

Inequality properties of real numbers

For all real numbers a, b, and c

Trichotomy property
Exactly one of the following is true.
$a < b$, $a = b$, or $a > b$

Transitive property of inequality
If $a < b$ and $b < c$, then $a < c$

■ Example 1–3 B

The following statements are examples of the properties of inequality.

1. $a \not< b$. The statement is read "a is not less than b." From the trichotomy property, we know that if a is not less than b, then a could be equal to b or a could be greater than b. We write this mathematically as $a \geq b$.
2. If $a < b$ and $b < 3$, then $a < 3$. — Transitive property ■

The following properties of real numbers govern the operations of addition and multiplication.

Properties of real numbers

For all real numbers a, b, and c

Closure property of addition
$a + b \in R$

Closure property of multiplication
$ab \in R$

Commutative property of addition
$a + b = b + a$

Commutative property of multiplication
$ab = ba$

Associative property of addition
$(a + b) + c = a + (b + c)$

Associative property of multiplication
$(ab)c = a(bc)$

Identity property of addition
There is a unique real number 0 such that
$a + 0 = 0 + a = a$ Zero is called the additive identity.

Identity property of multiplication
There is a unique real number 1 such that
$a \cdot 1 = 1 \cdot a = a$ One is called the multiplicative identity.

Additive inverse property
For every real number a, there is a unique real number $-a$ (read "the opposite of a" or "the negative of a" or "the additive inverse of a") such that
$a + (-a) = 0$ and $(-a) + a = 0$

Multiplicative inverse property (reciprocal)
For every real number a, $a \neq 0$, there is a unique real number $\frac{1}{a}$ such that

$a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$

Distributive property of multiplication over addition
$a(b + c) = ab + ac$

Example 1–3 C

The following statements are applications of the properties of real numbers.

1. $5 + 4 = 4 + 5$ Commutative property of addition
   $9 = 9$
2. $3 \cdot 6 = 6 \cdot 3$ Commutative property of multiplication
   $18 = 18$
3. $(2 + 3) + 4 = 2 + (3 + 4)$ Associative property of addition
   $5 + 4 = 2 + 7$
   $9 = 9$

4. $(2 \cdot 3)4 = 2(3 \cdot 4)$
$6 \cdot 4 = 2 \cdot 12$
$24 = 24$
Associative property of multiplication

5. $5 + 0 = 5$ Identity property of addition

6. $6 \cdot 1 = 6$ Identity property of multiplication

7. $(-7) + 7 = 0$ Additive inverse property

8. $6 \cdot \frac{1}{6} = 1$ Multiplicative inverse property

9. $2(3 + 4) = (2 \cdot 3) + (2 \cdot 4)$
$2(7) = 6 + 8$
$14 = 14$
Distributive property

10. $3(8 - 10) = 3[8 + (-10)]$
$3(-2) = 3 \cdot 8 + 3(-10)$
$-6 = 24 + (-30)$
$-6 = -6$
Distributive property

11. $(-8) + (-4) \in R$
$-12 \in R$
Closure property of addition

12. $(-10) \cdot 3 \in R$
$-30 \in R$
Closure property of multiplication

There are many other properties of real numbers that we have not listed. These other properties, called **theorems,** are implied or can be shown to be true from the previous properties and definitions. A theorem is a property that we can prove is true. We will now state several theorems that follow logically from the properties already listed.

Addition property of equality

For all real numbers a, b, and c, if $a = b$, then

$$a + c = b + c$$

Concept

We can add the same quantity to both members (also called sides) of an equality without altering the equality.

To prove that this theorem is true for all real numbers, we must start with the given information $a = b$, and, by applying the properties of real numbers, show that the desired result, $a + c = b + c$, is true.

Steps	*Reasons*
1. a, b, and c are real numbers	Given
2. $a + c$ is a real number	Closure property of addition
3. $a + c = a + c$	Reflexive property of equality
4. $a = b$	Given
5. $a + c = b + c$	Substitution property of equality

To discuss the steps in the previous proof, we start with the given information that a, b, and c are real numbers and because of the closure property of addition, $a + c$ must be a real number. We use the reflexive property of equality to write the equality $a + c = a + c$. Finally, since it was given that $a = b$, the substitution property of equality allows us to replace a with b in the right member and finish the proof.

The proofs of the following theorems are provided in the exercise set, where you will be asked to supply the reasons.

Multiplication property of equality

For all real numbers a, b, and c, if $a = b$, then

$$a \cdot c = b \cdot c$$

Concept

We can multiply both members of an equality by the same number without altering the equality.

Zero factor property

For any real number a,

$$a \cdot 0 = 0 \text{ and } 0 \cdot a = 0$$

Concept

The product of zero and any number is zero.

Double-negative property

For any real number a,

$$-(-a) = a$$

Concept

The opposite of the opposite of any real number is the given number.

Example 1–3 D

The following statements are applications of the previous theorems.

1. If $a = 3$, then $a + 4 = 3 + 4$ — Addition property of equality
   $a + 4 = 7$
2. If $a = 5$, then $4 \cdot a = 4 \cdot 5$ — Multiplication property of equality
   $4a = 20$
3. $4 \cdot 0 = 0$ — Zero factor property
4. $-(-4) = 4$ — Double-negative property

Mastery points

Can you

- Apply the equality properties for real numbers?
- Apply the inequality properties for real numbers?
- Apply the properties of real numbers?

Exercise 1–3

Apply the indicated property to write a new expression that is equal to the given expression. Assume that all variables represent real numbers. See example 1–3 C.

Examples Commutative, $5 + y$ Distributive, $3(x + y)$

Solutions $y + 5$ Change the order $3x + 3y$ Distribute multiplication

1. Commutative, $4x + 3y$
2. Distributive, $4(a - b)$
3. Associative, $(4 + 2) + 6$
4. Identity, $4 + 0$
5. Commutative, $5a$
6. Distributive, $12(3 - y)$
7. Associative, $(2x)y$
8. Identity, $5 \cdot 1$
9. Inverse, $(-4) + 4$
10. Inverse, $4 \cdot \left(\frac{1}{4}\right)$

Replace each question mark with the correct symbol to make the given statement an application of the given property.

Example If $b = 4$ and $b + 3 = c$, then $? + 3 = c$; substitution property of equality.

Solution $4 + 3 = c$ Replace b with 4

11. $x + 5 = ?$; reflexive property of equality.
12. Either $x < 0$, $x = 0$, or $x > ?$; trichotomy property.
13. If $2a - 3 = x$, then $x = ?$; symmetric property of equality.
14. If $a = b$ and $b = 5$, then $a = ?$; transitive property of equality.
15. If $a = 7$ and $b + 5 = a$, then $b + 5 = ?$; substitution property of equality.
16. If $5 < x$ and $x < y$, then $? < y$; transitive property of inequality.

Rewrite each relation without the slash mark to obtain an equivalent statement. See example 1–3 B.

17. $a \not< 5$
18. $3 \not\leq b$
19. $a \not\geq b$
20. $b \not> c$
21. $x \neq 5$
22. $4 \not< y$ (boxed: $\boxed{22.}$)
23. $a \not\leq 6$
24. $b \not\geq 4$

Identify which property is being used. See example 1–3 C.

Examples $5 + a = a + 5$ $(-7) + 7 = 0$

Solutions Commutative property of addition Additive inverse property

25. $6 + 7$ is a real number.
26. $8(a + b) = 8a + 8b$
27. $8 \cdot 1 = 8$
28. $5(x - y) = (x - y)5$
29. $\frac{1}{4} \cdot 4 = 1$
30. $(-4) + 4 = 0$
31. $0 + 9 = 9$
32. $(ab)(cd) = (cd)(ab)$
33. $(6 + 7) + 8 = 6 + (7 + 8)$
34. $(6 \cdot 7)8 = 6(7 \cdot 8)$
35. $(2x)y = 2(xy)$
36. $3 + (2 + b) = (2 + b) + 3$
37. $14 + 0 = 14$
38. $\left(\frac{2}{3}\right)\left(\frac{3}{2}\right) = 1$
39. $x + 6 = 6 + x$
40. $4 \cdot 5$ is a real number.
41. $(5 + 4) + y = 5 + (4 + y)$
42. $x + (-x) = 0$
43. $(a + b) + 0 = (a + b)$
44. $5(2 + b) = 10 + 5b$

Fill in the missing values. Assume that all variables represent nonzero real numbers.

	Number	Additive inverse	Multiplicative inverse
Example		-4	
Solution	4		$\frac{1}{4}$
45.	6		
46.	10		
47.		5	
48.		2	
49.	x		
50.	y		
51.			$\frac{1}{7}$
52.			$\frac{1}{-5}$
53.		-3	
54.		-8	

Give a reason for each step in the following proofs.

55. Prove: For all real numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$.

a, b, and c are real numbers — a. ______
$a \cdot c$ is a real number — b. ______
$a \cdot c = a \cdot c$ — c. ______
$a = b$ — d. ______
$a \cdot c = b \cdot c$ — e. ______

56. Prove: For any real number a, $a \cdot 0 = 0 \cdot a = 0$.

a is a real number — a. ______
$a = a \cdot 1$ — b. ______
$a = a(1 + 0)$ — c. ______
$a = a \cdot 1 + a \cdot 0$ — d. ______
$a = a + a \cdot 0$ — e. ______
$0 = a \cdot 0$ — f. ______
$a \cdot 0 = 0$ — g. ______
$0 \cdot a = 0$ — h. ______

57. Prove: For any real number a, $-(-a) = a$.

a is a real number — a. ______
$-a + a = 0$ — b. ______
$-a + [-(-a)] = 0$ — c. ______
$a = -(-a)$ — d. ______

1–4 ■ Order of operations

When we perform several different arithmetic operations within an expression or a formula, we need a standard order in which the operations will be performed. Consider the following example.

If a woman deposits \$1,000 in a savings account at 6% simple interest, the amount (A) in her account after 1 year can be found by evaluating

$$A = 1{,}000 + 1{,}000(0.06)$$

More than one answer is possible depending on the order in which we perform the operations. For example, if we first add the 1,000 and 1,000 and then multiply by (0.06), we have

$$1{,}000 + 1{,}000(0.06) = 2{,}000(0.06) = 120$$

However if we multiply the 1,000 and (0.06) and to this product add the first 1,000 we have

$$1{,}000 + 1{,}000(0.06) = 1{,}000 + 60 = 1{,}060^*$$

To standardize the answer, we agree to the following order of operations, or priorities:

Order of operations, or priorities

1. **Groups** Perform any operations within a grouping symbol such as ( ) parentheses, [ ] brackets, { } braces, | | absolute value, or in the numerator or the denominator of a fraction.
2. **Exponents** Perform operations indicated by exponents.
3. **Multiply and divide** Perform multiplication and division in order from left to right.
4. **Add and subtract** Perform addition and subtraction in order from left to right.

Note

a. Within a grouping symbol, the order of operations will still apply.
b. If there are several grouping symbols intermixed, remove them by starting with the innermost one and working outward.

Note Within any priority, every operation is of equal importance (that is, multiplication is no more important than division in priority 3). Therefore when performing a particular priority, we start at the left and proceed to the right, doing only those operations within the priority that we are performing, as we come to them, without skipping around.

■ Example 1–4 A

Perform the indicated operations in the proper order and simplify.

1. $9 + 12 \cdot 3 \div 2 = 9 + 36 \div 2$ Priority 3, multiply
$= 9 + 18$ Priority 3, divide
$= 27$ Priority 4, add

2. $(19 - 1) \div 2 + 3 \cdot 4 = 18 \div 2 + 3 \cdot 4$ Priority 1, parentheses
$= 9 + 12$ Priority 3, divide and multiply
$= 21$ Priority 4, add

*This is the correct answer.

3. $\frac{1}{4} + \frac{3}{4} \div \frac{5}{8} = \frac{1}{4} + \frac{3}{\cancel{4}_1} \cdot \frac{\cancel{8}^2}{5}$ Invert and multiply

$= \frac{1}{4} + \frac{6}{5}$ Priority 3, multiply

$= \frac{5}{20} + \frac{24}{20}$ Least common denominator

$= \frac{5 + 24}{20}$ Priority 4, add in the numerator

$= \frac{29}{20} \text{ or } 1\frac{9}{20}$

4. $2^2 \cdot 5 - 6 \cdot 4 = 4 \cdot 5 - 6 \cdot 4$ Priority 2, exponent

$= 20 - 24$ Priority 3, multiply

$= -4$ Priority 4, subtract

5. $\frac{3}{4} - \frac{\cancel{3}^1}{\cancel{4}_1} \cdot \frac{\cancel{4}^1}{\cancel{9}_3} = \frac{3}{4} - \frac{1}{3}$ Priority 3, multiply

$= \frac{9}{12} - \frac{4}{12}$ Least common denominator

$= \frac{9 - 4}{12}$ Priority 4, subtract in the numerator

$= \frac{5}{12}$

6. $(8.21 + 12.43) \div 2.4 - (6.1)(4.7)$

$= (20.64) \div 2.4 - (6.1)(4.7)$ Priority 1, parentheses

$= 8.6 - 28.67$ Priority 3, divide and multiply

$= -20.07$ Priority 4, subtract

7. $\left(\frac{1}{3} + \frac{5}{8}\right) \div \frac{5}{6} = \left(\frac{8}{24} + \frac{15}{24}\right) \div \frac{5}{6}$ Least common denominator

$= \frac{8 + 15}{24} \div \frac{5}{6}$ Priority 1, add the fractions

$= \frac{23}{24} \div \frac{5}{6}$ Add the numerators

$= \frac{23}{\cancel{24}_4} \cdot \frac{\cancel{6}^1}{5}$ Invert and multiply

$= \frac{23}{20} \text{ or } 1\frac{3}{20}$ Priority 3

8. $(9.3)^2 - 3(2.1)(4.6)$

$= 86.49 - 3(2.1)(4.6)$ Priority 2, exponent

$= 86.49 - 28.98$ Priority 3, multiply

$= 57.51$ Priority 4, subtract

9. $\frac{3(2+4)}{11-8} - \frac{4+6}{2} = \frac{3(6)}{11-8} - \frac{4+6}{2}$ Priority 1, parentheses

$= \frac{18}{3} - \frac{10}{2}$ Priority 1, numerators and denominators

$= 6 - 5$ Priority 3, divide

$= 1$ Priority 4, subtract

10. $5[4 + 3(10 - 12)]$ We first simplify within the grouping symbol, starting with the innermost one

$= 5[4 + 3(-2)]$ Parentheses

$= 5[4 + (-6)]$ Multiply inside the brackets

$= 5[-2]$ Add inside the brackets

$= -10$ Multiply

▶ ***Quick check*** $6 + 5(7 - 3) - 2^2$ ■

Problem solving

Solve the following word problems using the order of operations.

■ **Example 1–4 B**

Choose a letter to represent the unknown quantity and find its value by performing the indicated operations.

1. Mrs. Miyazaki purchased 8 cans of oil at \$1.10 per can and 2 air filters at \$3.75 each. What was her total bill?

   Let x = Mrs. Miyazaki's total bill. 8 cans at \$1.10 per can cost $8 \cdot \$1.10$ and 2 air filters at \$3.75 each cost $2 \cdot \$3.75$. The total bill is given by

   $$x = 8 \cdot 1.10 + 2 \cdot 3.75$$
   $$= 8.80 + 7.50 = 16.30$$

   Mrs. Miyazaki's total bill was \$16.30

2. A woman worked a 40-hour week at \$8 per hour. If she worked 6 hours overtime at time and a half, how much money did she receive for the 46 hours of work?

   Let W = the woman's total wages for the week. 40 hours at \$8 per hour is $40 \cdot \$8$ and 6 hours at time and a half is $6 \cdot 1\frac{1}{2} \cdot \$8$. Thus

   $$W = 40 \cdot 8 + 6 \cdot 1\frac{1}{2} \cdot 8$$
   $$= 320 + 72$$
   $$= 392$$

   The woman's total wages for the week was \$392. ■

Mastery points

Can you

- Simplify expressions according to the order of operations?

Exercise 1–4

Perform the indicated operations and simplify. See example 1–4 A.

Example $6 + 5(7 - 3) - 2^2$

Solution

$= 6 + 5(4) - 2^2$	We first simplify within the grouping symbol
$= 6 + 5(4) - 4$	Perform the indicated power
$= 6 + 20 - 4$	Carry out the multiplication
$= 26 - 4$	Perform the addition
$= 22$	Subtract

1. $(-3) \cdot 4 + 14$

2. $3 \cdot 4 + (-8)$

3. $5 \cdot 6 - (-7)$

4. $6 - (-4)(3)$

5. $12 - (-5)(-4)$

6. $\dfrac{8 + 4}{6} + 5$

7. $\dfrac{18 - 6}{2} + 7 \cdot 5$

8. $3 - 12 \cdot \left(\dfrac{1}{4}\right)$

9. $5 - 15 \cdot \left(\dfrac{1}{3}\right)$

10. $8 + 0(5 - 7)$

11. $10 - 0(8 - 4)$

12. $8 + 6 - 4 \cdot 3 + 12$

13. $10 + 10 \div 10 - 10$

14. $14 \div 7 - 2 + 8 \cdot 4$

15. $24 \div 6 + 6 - 4 \cdot 3$

16. $9 - 18 \div 2 + 7 - 3^2$

17. $14 + 27 \div 3 \cdot 4 - 5^2$

18. $4 - (8 - 6)^2 + 3 \cdot 5$

19. $(4 - 9)^2 \cdot 2 + 7 - 4^2$

20. $12 + 3 \cdot 16 \div 4^2 - 5$

21. $18 - 3^2 \cdot 4 \div 6 + 7$

22. $8 - (12 - 9)^2 \cdot 2 + 5$

23. $26.4 - (3.7)(4.6)$

24. $\dfrac{2}{3} \div \left(\dfrac{5}{6} - \dfrac{4}{9}\right)$

25. $\dfrac{3}{8} - \dfrac{1}{2} \div \dfrac{3}{4}$

26. $\dfrac{1}{8} + \dfrac{7}{12} \cdot \dfrac{9}{14}$

27. $(14.13 + 11.4) \div 3.7 - (3.6) \cdot (4.9)$

28. $(4.6 + 3.1) \cdot (2.7) - (5.4) \cdot (7.3)$

29. $(4.7)^2 \cdot 5 - (14.64) \div (6.1)$

30. $(2.4)^2 + 5(1.9)^2 - 12.6$

31. $\left(\dfrac{11}{12} - \dfrac{5}{6}\right) \div \left(\dfrac{1}{3} + \dfrac{3}{8}\right)$

32. $\left(\dfrac{1}{4} - \dfrac{1}{6}\right) \cdot \left(\dfrac{1}{8} + \dfrac{3}{8}\right)$

33. $5[20 - 3(4 - 6) + 5]$

34. $9 + 2[5 - 3(8 - 3) + 5]$

35. $12 + [14 - 5(7 - 10) + 4]$

36. $(9 - 7)[15 - 4(3 - 6) + 9]$

37. $\dfrac{8(6 - 4)}{2} - \dfrac{27}{-3}$

38. $\dfrac{3(9 - 4)}{5} - \dfrac{10}{-2}$

39. $\dfrac{5 \cdot 6 - 4}{13} - \dfrac{(-18)}{6}$

40. $\dfrac{8 - 3 \cdot 4 + 10}{6 - 3} + \dfrac{(-18) + 6}{-4}$

41. $\left[\dfrac{10 + (-2)}{2(-1)}\right]\left[\dfrac{(-10)(-4)}{(-2)}\right]$

42. $\left[\dfrac{(-11) + (-7)}{(-9)}\right]\left[\dfrac{(-5)(-12)}{10}\right]$

43. $\dfrac{4(3 + 2) - 4^2 + 11}{(-5)(-3)}$

44. $\dfrac{(8 - 4)^2 + (-2)(-3)}{10 - (-4)(3)}$

Perform the indicated operations and simplify. See example 1–4 A.

45. The area of a trapezoid whose bases are 12 meters and 8 meters and whose height is 10 meters is

$$\frac{1}{2} \cdot 10(12 + 8)$$

Find the area in square meters.

46. The surface area of a flat ring whose inside radius is 3 centimeters and whose outside radius is 5 centimeters is approximately

$$\frac{22}{7} \cdot 5^2 - \frac{22}{7} \cdot 3^2$$

Find the area in square centimeters.

47. The water pressure exerted on 1 square foot (144 square inches) of surface area of a diving suit 50 feet below the surface of the water is given by

$$144\,[14.7 + 50(0.444)]$$

Perform the indicated operations to find this pressure in pounds per square inch.

48. The surface area of a ring section whose inside diameter is 19 inches and whose outside diameter is 26 inches is approximately

$$\frac{22}{7} \cdot \frac{26 + 19}{2} \cdot \frac{26 - 19}{2}$$

Find the surface area in square inches.

49. The surface area of a rectangular solid whose length is 8 feet, width is 5 feet, and height is 7 feet is given by

$$2 \cdot 8 \cdot 7 + 2 \cdot 8 \cdot 5 + 2 \cdot 5 \cdot 7$$

Find the surface area in square feet.

50. To convert 59 degrees Fahrenheit to degrees Celsius, we use

$$\frac{5}{9}\,(59 - 32)$$

Find the temperature in degrees Celsius.

51. To convert 25 degrees Celsius to degrees Fahrenheit, we use

$$\left(\frac{9}{5}\right)25 + 32$$

Find the temperature in degrees Fahrenheit.

Choose a letter to represent the unknown quantity and solve. See example 1–4 B.

52. Norbert makes \$6 per hour for a 40-hour week and receives time and a half for every hour he works over 40 hours a week. How much did he earn if he worked 47 hours in one week?

53. Royetta has \$120 in her checking account. She deposits three twenty-dollar checks and then writes a check for \$87. What is her balance?

54. A man purchased a case of beans (12 cans) at 53¢ per can, 4 pounds of apples at 59¢ per pound, and 3 quarts of milk at 69¢ per quart. What was his total bill (a) in cents, (b) in dollars and cents?

55. A car dealer sold 18 cars at \$8,200 each and 4 trucks at \$7,100 each. What is the total sale of all the cars and trucks?

56. Forest Road School has 3 first-grade classes, each with 27 students; 2 second-grade classes, each with 31 students; and 2 third-grade classes, each with 29 students. How many students are there all together in first, second, and third grade?

57. While away at school for nine months, a student spends \$140 per month on housing, \$95 per month on food, and \$48 per month on miscellaneous expenses. How much money did the student spend on these expenses during the nine months?

58. A woman wants to carpet two rooms. One room is 4 yards by 5 yards and the second room is 4 yards by 7 yards. How many square yards of carpet are needed? If the carpet costs \$8 per square yard and the pad under the carpet costs \$2 per square yard, how much will the carpet and pad for these rooms cost?

59. Nancy, Alice, and Jane are typists in an office. If Nancy can type 90 words per minute, Alice can type 85 words per minute, and Jane can type 70 words per minute, how many words can they type together in 20 minutes?

60. Susan charges \$5 to wash a car, \$11 to mow a lawn, and \$27 to wax a car. If in one week, she washed six cars, mowed three lawns, and waxed two cars, how much did she earn that week?

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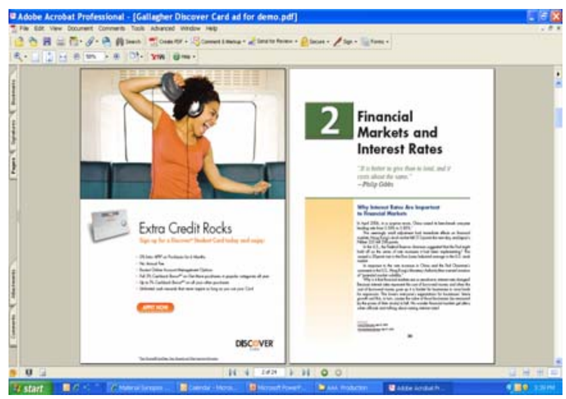

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1–5 ■ Terminology and evaluation

Terminology

Before we study the different types of algebraic operations, we must first define a few concepts. *An* **algebraic expression** *is any meaningful collection of variables, constants, grouping symbols, and symbols of operations.* Examples of algebraic expressions are

$$3x^2y,\ \frac{ab}{c},\ \pi r^2,\ \frac{x^2+2x-1}{x^2+1},\ 2x^2-x+3,\ \sqrt{x^2+y^2},\ x,\ -4$$

In an algebraic expression, *a* **term** *is any constant, variable, or indicated product, quotient, or root* of constants and variables.* Terms in an algebraic expression are separated by the operations of addition and subtraction.

Note *A constant is a symbol that does not change its value.* In the expression πr^2, π is a symbol that represents only one value (the circumference of a circle divided by its diameter equals 3.14159 · · ·) and therefore is a constant.

■ Example 1–5 A

Identify the number of terms in the algebraic expressions.

The plus and minus signs separate the algebraic expression into three terms

1. $\underset{\text{1st}}{3x^2} - \underset{\text{2nd}}{2x} + \underset{\text{3rd}}{1}$ There are three terms

2. $\underset{\text{1st}}{3x^2} - \underset{\text{2nd}}{\dfrac{y^2-3z^3}{5x}}$ There are two terms since the fraction bar acts as a grouping symbol

▶ ***Quick check*** Identify the number of terms in $x^2 + 2y^2$. ■

In the expression $3xy$, *each factor or group of factors is called the* **coefficient** *of the remaining factors.* That is, 3 is the coefficient of xy, x is the coefficient of $3y$, $3x$ is the coefficient of y, and so on. The 3 is called the **numerical coefficient,** and it tells us how many xy's we have in the expression.

Since we most often talk about the numerical coefficient of a term, we will eliminate the word "numerical" and just say "coefficient." It will be understood that we are referring to the numerical coefficient. If no numerical coefficient appears in a term, the coefficient is understood to be 1.

■ Example 1–5 B

Determine the numerical coefficient of each term in the algebraic expression $3x^2 - 2y + z$.

3 is the coefficient of x^2, -2 is the coefficient of y, and 1 is understood to be the coefficient of z.

Note The coefficient of a term in an algebraic expression includes the sign that precedes it. In our example, the coefficient of y is -2 and not 2.

*Roots will be discussed in chapter 5.

▶ ***Quick check*** Determine the numerical coefficient of each term in $3a^3 - 4a^2 + 2a$. ■

A special kind of algebraic expression is a *polynomial*. The following are characteristics of a polynomial.

1. It has real number coefficients.
2. All variables in a polynomial are raised only to natural number exponents.
3. The operations performed by the variables are limited to addition, subtraction, and multiplication.

A polynomial that contains just one term is called a *monomial;* one that contains two terms is called a *binomial;* and a polynomial that contains three terms is called a *trinomial*. Any polynomial that contains more than one term is called a *multinomial*, but no special names will be given to polynomials that contain more than three terms.

Note We should simplify any expression before identifying it. Also, in an expression, the combining of all the constant terms is understood to be a single term. For example, $x + 3 + \pi$ is thought of as $x + (3 + \pi)$ and is a binomial.

■ **Example 1–5 C**

Determine if the algebraic expressions are polynomials.

1. x^2, $5x$, 7, and $-4x^2y^3$ are monomials.
2. $2x - 4$, $x^2 + y^2$, and $27x^3 - y^3$ are binomials.
3. $4x^2 + 3x - 2$ and $6x^2y^2 - 4xy + 3y^2$ are trinomials.
4. $8x^3 - 4x^2 + 3x - 2$ has no special name and is referred to as a polynomial of four terms.
5. $\dfrac{x}{y + z}$ and $5\sqrt{x} + y$ are not polynomials since they contain variables in the denominator or under a radical symbol. ■

We also identify different types of polynomials by the degree of the polynomial. *The* **degree** *of a polynomial in one variable is the greatest exponent of that variable in any one term.*

■ **Example 1–5 D**

Determine the degree of the polynomial.

1. $5x^3$ Third degree because the exponent of x is 3
2. $x^4 - 2x^3 + 3x - 5$ Fourth degree because the greatest exponent of x in any one term is 4

Note In example 2, the polynomial has been arranged in *descending powers* of the variable. This is the form that we will use when we write polynomials in one variable.

▶ ***Quick check*** Identify $2x^2 - x + 5$ as a monomial, binomial, or trinomial and determine the degree. ■

Evaluating an algebraic expression

An extremely important process in algebra is that of calculating the numerical value of an expression when we are given specific replacement values for the variables. This process is called **evaluation.** To carry out this evaluation, we need the following **property of substitution.**

Property of substitution

If $a = b$, then a may be replaced by b or b may be replaced by a in any expression without altering the value of the expression.

Concept

When two quantities are equal, we can replace one quantity with the other quantity without altering the value of the expression.

We frequently need to evaluate algebraic expressions. For example, the area A of a rectangle is found by multiplying the length ℓ times the width w, $A = \ell w$. If the length is 8 feet and the width is 4 feet, substituting 8 for the length, ℓ, and 4 for the width, w, the expression becomes

$$A = (8)(4) = 32$$

and the area is 32 square feet. We substituted the respective values for the length and the width into the expression and carried out the indicated arithmetic.

Note When replacing variables with the numbers they represent, it is a good procedure to put each of the numbers inside parentheses.

Example 1–5 E

Evaluate the following expressions for the given real number replacement for the variable or variables.

1. $x^2 + 3x - 4$, when $x = 3$

$= (\quad)^2 + 3(\quad) - 4$	Expression without the x
$= (3)^2 + 3(3) - 4$	Substitute 3 in place of each x and use the order of operations
$= 9 + 3(3) - 4$	Exponents
$= 9 + 9 - 4$	Multiply
$= 14$	Add and subtract

Therefore the expression $x^2 + 3x - 4$ evaluated for $x = 3$ is 14.

2. If we know the temperature in degrees Fahrenheit (F), the temperature in degrees Celsius (C) can be found by the formula $C = \frac{5}{9}(F - 32)$. Find the temperature in degrees Celsius if the temperature is 77 degrees Fahrenheit.

$C = \frac{5}{9}(F - 32)$	Original formula
$C = \frac{5}{9}[(\quad) - 32]$	Formula ready for substitution
$C = \frac{5}{9}[(77) - 32]$	Substitute
$C = \frac{5}{9}[45]$	Order of operations
$C = 25$	

Hence 77 degrees Fahrenheit is equivalent to 25 degrees Celsius.

3. If a company sells n_1 shirts at d_1 dollars each and n_2 ties at d_2 dollars each, the total revenue R from the shirts and ties would be expressed as $R = n_1d_1 + n_2d_2$. Find R if $n_1 = 20$, $d_1 = 26$, $n_2 = 15$, and $d_2 = 8$.

Note In this formula, **subscripts** are used to denote two different values for the number of articles sold (n_1 and n_2) and the price per article (d_1 and d_2). These are read "n sub-one," "n sub-two," "d sub-one," and "d sub-two."

$R = n_1d_1 + n_2d_2$	Original formula
$R = (\ )(\ ) + (\ )(\ )$	Formula ready for substitution
$R = (20)(26) + (15)(8)$	Substitute
$R = 520 + 120$	Order of operations
$R = 640$	

Therefore the total revenue from the sale of shirts and ties is \$640.

▶ *Quick check* Find the temperature C for $C = \frac{5}{9}(F - 32)$ when $F = 95$. ■

Polynomial notation

Polynomials can be denoted by a single symbol such as P, Q, R, or by P_1, P_2, P_3, and so on. If we are dealing with a polynomial in one variable, the symbol for the polynomial can be used in conjunction with the variable in naming the polynomial. For example, $P(x)$ (read "P of x" or "P at x") could be used to denote the polynomial $x^2 + 3x - 4$ from number 1 of example 1–5 E. Therefore we can write $P(x) = x^2 + 3x - 4$.

The symbol inside the parentheses denotes the variable in the polynomial. We can use this notation to indicate a specific value at which we want to evaluate the polynomial. In example 1–5 E, we evaluated $x^2 + 3x - 4$ at 3. We could have represented this as $P(3)$, which means the polynomial P evaluated at 3, not P times 3.

■ ***Example 1–5 F***

If $P(x) = x^2 - x + 6$ and $Q(y) = y + 2$, find the indicated values.

1. $Q(-5)$

$Q(\ \ ) = (\ \ ) + 2$	Polynomial ready for substitution
$Q(-5) = (-5) + 2$	Substitute -5 for y in $Q(y)$
$= -3$	

2. $P(-3)$

$P(\ \ ) = (\ \ )^2 - (\ \ ) + 6$	Polynomial ready for substitution
$P(-3) = (-3)^2 - (-3) + 6$	Substitute -3 for x in $P(x)$
$= 9 - (-3) + 6$	
$= 18$	

3. $P(-4) \cdot Q(-6)$

Substitute -4 for x in $P(x)$ and -6 for y in $Q(y)$.

$= [(-4)^2 - (-4) + 6] \cdot [(-6) + 2]$	Substitute
$= [16 - (-4) + 6] \cdot [-4]$	Order of operations
$= [26] \cdot [-4]$	$P(-4) = 26$ and $Q(-6) = -4$
$= -104$	Multiply

Note $P(-4)$ and $Q(-6)$ represent specific real numbers and we must evaluate each of them at the indicated value before we can perform the indicated multiplication.

▶ ***Quick check*** If $P(x) = x^2 - x + 6$, find $P(3)$.

Algebraic notation

Many problems that we encounter are stated verbally. These need to be translated into algebraic expressions. While there is no standard procedure for changing a verbal phrase into an algebraic expression, these guidelines should be of use.

1. Read the problem carefully, determining useful prior knowledge. Note what information is given and what information we are asked to find.
2. Let some letter represent one of the unknowns. Then express any other unknowns in terms of it.
3. Use the given conditions in the problem and the unknowns from step 2 to write an algebraic expression.

When translating verbal phrases into algebraic expressions, we look for phrases that involve the basic operations of addition, subtraction, multiplication, and division. Table 1–1 shows some examples of phrases that we commonly encounter. We let x represent the unknown number.

Table 1–1

Phrase	Algebraic expression
Addition	
6 more than a number the sum of a number and 6 6 plus a number a number increased by 6 6 added to a number	$x + 6$
Subtraction	
6 less than a number a number diminished by 6 the difference of a number and 6 a number minus 6 a number less 6 a number decreased by 6 6 subtracted from a number a number reduced by 6	$x - 6$
Multiplication	
a number multiplied by 6 6 times a number the product of a number and 6	$6x$
Division	
a number divided by 6 the quotient of a number and 6 $\frac{1}{6}$ of a number	$\frac{x}{6}$

■ Example 1–5 G

Write an algebraic expression for each of the following.

1. The product of x and y — $x \cdot y$
2. The sum of n and 7 — $n + 7$
3. y decreased by 3 — $y - 3$
4. a divided by 4 — $a \div 4$ or $\frac{a}{4}$
5. A number diminished by 6, let n represent the number. — $n - 6$
6. Three times a number and that product increased by 9, let x represent the number. — $3x + 9$
7. A number divided by 4 and that quotient decreased by 2, let a represent the number. — $\frac{a}{4} - 2$
8. Five times the sum of a number and 6, let n represent the number. — $5(n + 6)$

▶ ***Quick check*** Write an algebraic expression for each of the following:
1. the product of a and b
2. a number increased by 2 ■

Mastery points

Can you

- Identify terms in an expression?
- Identify a polynomial by name?
- Determine the degree of a polynomial?
- Evaluate an algebraic expression?
- Evaluate a polynomial using polynomial, $P(x)$, notation?
- Write an algebraic expression?

Exercise 1–5

Specify the number of terms in each expression and determine if the expression is a polynomial. See examples 1–5 A and C.

Example $x^2 + 2y^2$

Solution $\underset{\substack{\uparrow \\ \text{1st}}}{x^2} + \underset{\substack{\uparrow \\ \text{2nd}}}{2y^2}$ There are two terms; is a polynomial (binomial)

1. $5x^2 - 2x + 3$
2. $\frac{7x}{2}$
3. $3xy - \frac{3y}{5} + 7x$
4. $\boxed{4.}$ $\frac{4a^2 - b^2}{10}$
5. $\sqrt{3x^2 - 2x + 1}$
6. $\frac{x^2 + y^2}{z^2} - a^2 + 2ab$

Determine the numerical coefficient of each term in the following expressions. See example 1–5 B.

Example $3a^3 - 4a^2 + 2a$

Solution The coefficient of a^3 is 3, the coefficient of a^2 is -4, and the coefficient of a is 2.

7. $4x^2 - 7x + y$
8. $\boxed{8.}$ $3x^3 - 2x^2 + x$
9. $5y^3 + y^2 - 7y$
10. $4z^4 - z^3 + z^2 + 6z$

Identify each polynomial as a monomial, binomial, or trinomial and determine the degree of the polynomial. See examples 1–5 C and D.

Example $2x^2 - x + 5$

Solution There are three terms, so it is a trinomial, and second degree because the greatest exponent of x in any one term is 2.

11. $5a^3 - 4a^2 + 3$
12. $4a^2 - a$
13. $2x^4 - 7x^2$
14. $6x^2 - 2x + 1$
15. $4y^5$

Evaluate the following expressions if $a = -3$, $b = 2$, $c = -2$, and $d = 4$. See example 1–5 E.

16. $2(a + 3b)$
17. $2a - 3b - (2c + d)$
18. $b - 3(a - 2d)$
19. $\boxed{19.}$ $(4a + 3b)(c - 2d)$
20. $5ab^2 + 3cd$
21. $2a^2 - 3a + 4$
22. $c^2 - d^2$
23. $a^2 - 4c^2$
24. $\boxed{24.}$ $(3a - 2b) - (2a + b)(c + d)$
25. $a^3b - c^3d$

Evaluate the following polynomials for the specific values of the variable. See example 1–5 F.

Example $P(x) = x^2 - x + 6$, find $P(3)$.

Solution

$= (3)^2 - (3) + 6$	Substitute 3 for x, order of operations
$= 9 - 3 + 6$	Exponents
$= 12$	Subtract and add

26. $P(x) = x^2 + x$, $P(3)$, $P(-2)$, $P(0)$
27. $Q(x) = x^2 - 3x + 4$, $Q(2)$, $Q(-3)$, $Q(0)$
28. $R(x) = 2x^2 - x - 1$, $R(-2)$, $R(1)$, $R(0)$
29. $P(y) = y^3 - 1$, $P(1)$, $P(-2)$, $P(0)$
30. $Q(z) = z^2 - 5z + 6$, $Q(3)$, $Q(2)$, $Q(0)$

If $P(x) = x^2 + 2x + 1$, $Q(x) = 2x - 1$, and $R(x) = x^2 + x - 6$, find the indicated values. See example 1–5 F.

Example $R[Q(2)]$

Solution

$= R[2(2) - 1]$	Substitute 2 for x in $Q(x)$
$= R[4 - 1]$	Multiply
$= R(3)$	Subtract
$= (3)^2 + (3) - 6$	Substitute 3 for x in $R(x)$
$= 9 + 3 - 6$	Exponents
$= 6$	Add and subtract

Note In this example, we had to evaluate the inner polynomial, $Q(x)$, first so that we could determine the value for which we would evaluate the outer polynomial, $R(x)$.

31. $P(2) + Q(-1) - R(2)$ **32.** $P(0) \cdot Q(5)$ **33.** $P(4) - R(0)$

34. $P(-1) \cdot Q(-1) + R(-1)$ **35.** $Q(4) \cdot R(-1)$ **36.** $P[Q(2)]$

37. $P[Q(-1)]$ **38.** $R[P(1)]$ **39.** $Q[R(-2)]$

Evaluate the following formulas. See example 1–5 E.

Example Find the temperature C for $C = \frac{5}{9}(F - 32)$ when $F = 95$.

Solution

$C = \frac{5}{9}[(\quad) - 32]$ Formula ready for substitution

$C = \frac{5}{9}[(95) - 32]$ Substitute

$C = \frac{5}{9}[63]$ Order of operations

$C = 35$

40. $V = e^3, e = 5$

41. $F = ma$, $m = 12$ and $a = 5$

42. $I = prt$; $p = 2{,}000$; $r = 0.09$; and $t = 3$

43. $V = k + gt$, $k = 22$, $g = 11$, and $t = 3$

44. $H = \frac{D^2N}{2}$, $D = 4$ and $N = 6$

45. $\ell = a + (n - 1)d$, $a = 7$, $n = 20$, and $d = 4$

46. $V_2 = V_1 + at$, $V_1 = 80$, $a = 4$, and $t = 6$

47. $S = \frac{1}{2}gt^2$, $g = 32$ and $t = 3$

48. $S = V_0t + \frac{1}{2}at^2$, $V_0 = 22$, $t = 4$, and $a = 32$

49. $A = \frac{n_1P_1 + n_2P_2}{n_1 + n_2}$, $n_1 = 80$, $P_1 = 3$, $n_2 = 110$, and $P_2 = 5$

50. $R_x = \frac{R_1R_3}{R_2}$, $R_1 = 8$, $R_2 = 5$, and $R_3 = 15$

51. $V_2 = \frac{V_1P_1}{P_2}$, $V_1 = 14$, $P_1 = 30$, and $P_2 = 35$

52. $V_1 = \frac{V_2P_2}{P_1}$, $V_2 = 12$, $P_1 = 22$, and $P_2 = 33$

53. $V_2 = \frac{V_1T_2}{T_1}$, $V_1 = 14$, $T_1 = 18$, and $T_2 = 27$

Evaluate the following formulas. See example 1–5 E.

54. The volume V of a rectangular solid is found by multiplying length ℓ times width w times height h, $V = \ell wh$. Find the volume in cubic feet if $\ell = 12$ feet, $w = 4$ feet, and $h = 5$ feet.

55. The perimeter P of a rectangle is found by the formula $P = 2\ell + 2w$, where ℓ is the length of the rectangle and w is the width. Find the perimeter of the rectangle in meters if $\ell = 12$ meters and $w = 7$ meters.

56. It is necessary to drag a box 600 feet across a level lot in 3 minutes. The force required to pull the box is 2,000 pounds. What is the horsepower (h) needed to do this if h is defined by $h = \frac{L \cdot W}{33{,}000 \cdot t}$, where L = distance to be moved, W = force exerted, and t = time in minutes required to move the box through L?

57. A pulley 12 inches in diameter that is running at 320 revolutions per minute (rpm) is connected by a belt to a pulley 9 inches in diameter. How many revolutions per minute will the smaller pulley make if $s = \frac{SD}{d}$, where s = speed of smaller pulley, d = diameter of smaller pulley, S = speed of larger pulley, and D = diameter of larger pulley?

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58. In a gear system, the velocity V of the driving gear is defined by $V = \frac{vn}{N}$, where v = velocity of the follower gear, n = number of teeth of the follower gear, and N = number of teeth of the driving gear. Find V when $v = 90$ rpm, $n = 30$ teeth, $N = 65$ teeth.

59. A formula in electricity is $I = \frac{E}{R}$, where I represents the current measured in amperes in a certain part of a circuit, E is the potential difference in voltage across that part of the circuit, and R is the resistance in ohms of that part of the circuit. Find I in amperes if $E = 110$ volts and $R = 44$ ohms.

Write an algebraic expression for each of the following. See example 1–5 G.

Examples The *product* of a and b — A *number increased by* 2

Solutions $a \cdot b$ — Let n represent the number; hence $n + 2$

60. The sum of x and 8

61. 3 times x

62. 5 less than n

63. 8 more than x

64. x increased by 5 and that sum divided by 6

65. 4 times the sum of x and 7

66. n decreased by 8

67. x decreased by 4

68. a divided by 3 and that quotient increased by 4

69. A number decreased by 15

70. A number added to 6

71. 8 times a number and that product increased by 14

72. A number divided by 5 and that quotient decreased by 2

73. 4 times the sum of a number and 3

74. A number decreased by 6 and that difference divided by 9

75. $\frac{1}{4}$ of a number

76. $\frac{4}{5}$ of a number

77. One-half of a number

78. One-third of a number

1–6 ■ Sums and differences of polynomials

In section 1–5, we learned by the process of evaluation that a polynomial is a symbol representing a real number. Therefore the ideas and properties that apply to operations with real numbers also apply to polynomials. Let us now examine the operations of addition and subtraction of polynomials.

An application of the distributive property

Using the symmetric property of equality and the commutative property, we can write the distributive property of multiplication over addition as

$$ax + bx = (a + b)x$$

Consider the expression

$$4x + 5x$$

Using the distributive property, the expression can be written

$$4x + 5x = (4 + 5)x = 9x$$

In this expression, $4x$ and $5x$ are terms that we wish to add. Terms are separated by the operations of addition and subtraction.

Simplifying expressions, as in this example, is *combining like terms.* **Like terms** are terms that may differ only in their numerical coefficients. For two or more terms to be called like terms, the variable factors of the terms along with their respective exponents must be identical. However the numerical coefficients may be different.

1. $5x^2y^3$, $-3x^2y^3$, and x^2y^3 are like terms because they differ only in their numerical coefficients.
2. $5a^3b^2$ and $5a^2b^3$ both contain the same variables but are not like terms because the exponents of the respective variables are not the same.

Combining like terms

1. Identify the like terms.
2. If necessary, use the commutative and associative properties to group together the like terms.
3. Combine the numerical coefficients of the like terms and multiply that by the variable factor.
4. Remember that y is the same as $1 \cdot y$ and $-y$ is the same as $-1 \cdot y$.

Note The process of addition or subtraction is performed only with the numerical coefficients, the variable factors remain unchanged.

Example 1–6 A

Perform the indicated addition or subtraction.

1. $3z - 7z + z - 6 + 15$ — Identify like terms

$= (3 - 7 + 1)z + (-6 + 15)$ — Associative and distributive properties

$= -3z + 9$ — Combine numerical coefficients, combine numbers

2. $3a^2b + 5a^2b + 2a^2b = (3 + 5 + 2)a^2b = 10a^2b$

3. $x^3y + \overbrace{4xy^2 - 3x^3y + 2xy^2}^{\text{Like terms}}$ — Identify any like terms

$\underbrace{x^3y + 4xy^2 - 3x^3y}_{\text{Like terms}}$

$= [1 + (-3)]x^3y + (4 + 2)xy^2$ — Commutative, associative, and distributive properties

$= -2x^3y + 6xy^2$ — Combine numerical coefficients

Note The numerical coefficient of a term includes the sign that precedes it. Therefore we consider any addition and subtraction of terms as a sum of terms in which the sign that precedes the term is taken as the sign of the numerical coefficient.

► ***Quick check*** Perform the indicated addition or subtraction.
$5xy^3 - 3xy^3 + 7xy^3$

Removing grouping symbols

We learned in chapter 1 that any quantity enclosed within a grouping symbol is treated as a single number. Now we are going to use the distributive property to remove grouping symbols such as ( ), [ ], and { }. Consider the following examples:

1. The quantity $(2x + y)$ can be written $1 \cdot (2x + y)$ because if there is no numerical coefficient, then 1 is understood to be the coefficient. Applying the distributive property,
$$1(2x + y) = 1 \cdot 2x + 1 \cdot y = 2x + y$$
2. The quantity $+ (2x + y)$ can be written $+1 \cdot (2x + y)$, giving
$$+1 \cdot (2x + y) = (+1) \cdot 2x + (+1) \cdot y = 2x + y$$
3. The quantity $-(2x + y)$ can be written $-1 \cdot (2x + y)$, giving
$$-1 \cdot (2x + y) = (-1) \cdot 2x + (-1) \cdot y = -2x - y$$

Removing grouping symbols

1. If a grouping symbol is preceded by no symbol or by a "+" sign, the grouping symbol can be dropped and the enclosed terms remain unchanged.
2. If a grouping symbol is preceded by a "−" sign, when the grouping symbol is dropped, we change the sign of each enclosed term.

Example 1–6 B

Remove all grouping symbols and perform the indicated addition or subtraction.

1. $(3x^2 + 2xy - 3y^2) + (x^2 - 5xy + 2y^2)$ — Remove grouping symbols

$= 3x^2 + 2xy - 3y^2 + x^2 - 5xy + 2y^2$ — Enclosed terms remain unchanged

$= (3x^2 + x^2) + (2xy - 5xy) + (-3y^2 + 2y^2)$ — Associative and commutative properties

$= 4x^2 - 3xy - y^2$ — Combine like terms

Note We can carry out the addition of these same two polynomials by lining them up in columns such that the like terms appear in the same column.

$$\begin{array}{r} (3x^2 + 2xy - 3y^2) \\ +\ \underline{(\ x^2 - 5xy + 2y^2)} \end{array} \qquad \begin{array}{r} 3x^2 + 2xy - 3y^2 \\ \underline{x^2 - 5xy + 2y^2} \\ 4x^2 - 3xy - y^2 \end{array}$$

2. $(2x^2 - 7x + 6) - (x^2 - 5x + 9)$ — Remove grouping symbols

$= 2x^2 - 7x + 6 - x^2 + 5x - 9$ — Change the sign of each term in the second parentheses

$= (2x^2 - x^2) + (-7x + 5x) + (6 - 9)$ — Commutative and associative properties

$= x^2 - 2x - 3$ — Combine like terms

Note This example may also be worked in the column form, but remember that when we remove the second pair of parentheses, we must change the sign of each enclosed term and then combine the like terms.

$$\begin{array}{rl} & (2x^2 - 7x + 6) \\ - & \underline{(\ x^2 - 5x + 9)} \end{array} \qquad \begin{array}{r} 2x^2 - 7x + 6 \\ \underline{-x^2 + 5x - 9} \\ x^2 - 2x - 3 \end{array}$$

3. $(5a^2 + 3a^2b - 3b^2) - (2a^2 - 3ab^2 - 4b^2)$ — Remove grouping symbols

$= 5a^2 + 3a^2b - 3b^2 - 2a^2 + 3ab^2 + 4b^2$ — Change the sign of each term in the second parentheses

$= (5a^2 - 2a^2) + 3a^2b + 3ab^2 + (-3b^2 + 4b^2)$ — Group like terms

$= 3a^2 + 3a^2b + 3ab^2 + b^2$ — Combine like terms

By the column method, we have

$$\begin{array}{rl} & (5a^2 + 3a^2b - 3b^2) \\ - & \underline{(2a^2 - 3ab^2 - 4b^2)} \end{array} \quad \rightarrow \quad \begin{array}{rl} & 5a^2 + 3a^2b - 3b^2 \\ - & \underline{2a^2 + 3ab^2 + 4b^2} \end{array}$$

$$\begin{array}{rlllll} & 5a^2 & + 3a^2b & & - 3b^2 \\ - & \underline{2a^2} & \underline{} & \underline{+ 3ab^2} & \underline{+ 4b^2} \\ & 3a^2 & + 3a^2b & + 3ab^2 & + b^2 \end{array}$$

Rearrange so that like terms appear in the same column

Note In future examples, we will mentally regroup the like terms and use the horizontal method to add or subtract polynomials.

► ***Quick check*** Remove all grouping symbols and perform the indicated addition or subtraction. $(5x^2 + 2x - 4) - (3x^2 - 7x + 9)$ ■

There are many situations in which there will be grouping symbols within grouping symbols. In these cases, *it is usually easier to remove the innermost grouping symbol first.*

■ Example 1–6 C

Remove all grouping symbols and perform the indicated addition or subtraction.

1. $2a^2 - [a + (a^2 - 3a)] = 2a^2 - [a + a^2 - 3a]$ — Remove parentheses first

$= 2a^2 - [a^2 - 2a]$ — Combine like terms within brackets

$= 2a^2 - a^2 + 2a$ — Remove brackets

$= a^2 + 2a$ — Combine like terms

After removing the parentheses, we combine the like terms before removing the brackets. *Simplify within a group whenever possible.*

2. $3x - \{6x - [y - (3x + 2y)] + 2x\}$

$= 3x - \{6x - [y - 3x - 2y] + 2x\}$ — Remove innermost grouping symbol

$= 3x - \{6x - [-y - 3x] + 2x\}$ — Combine like terms within brackets

$= 3x - \{6x + y + 3x + 2x\}$ — Remove brackets

$= 3x - \{11x + y\}$ — Combine like terms within braces

$= 3x - 11x - y$ — Remove braces

$= -8x - y$ — Combine like terms

► ***Quick check*** Remove all grouping symbols and perform the indicated addition or subtraction. $(5x^2 - 3x + 2) - [2x^2 - (x^2 - 7)]$ ■

Mastery points

Can you

- Identify like terms?
- Perform addition and subtraction of polynomials?
- Remove grouping symbols?

Exercise 1–6

Perform the indicated addition or subtraction. See example 1–6 A.

Example $5xy^3 - 3xy^3 + 7xy^3$ — Determine if we have like terms

Solution $= (5 - 3 + 7)xy^3$ — Distributive property

$= 9xy^3$ — Combine numerical coefficients

1. $4x + 6x^2 - 2x^2 + 3x - 5x^2 + x$
2. $5y^2 - 12y + 6y^5 - 4y^2 + y$
3. $-a^2 + a - 5a^3 + 2a^2 + 4a$
4. $2x^2y - 4xy^2 + 3x^2y$
5. $5x^2y - 3xy + 6xy - x^2y$
6. $6a^2b - 2ab^2 + 3a^2b - 4a^2b^2$

Remove all grouping symbols and perform the indicated addition or subtraction. See example 1–6 B.

Example $(5x^2 + 2x - 4) - (3x^2 - 7x + 9)$

Solution $= 5x^2 + 2x - 4 - 3x^2 + 7x - 9$ — Change the sign of each term in the second parentheses

$= (5x^2 - 3x^2) + (2x + 7x) + (-4 - 9)$ — Commutative and associative properties

$= 2x^2 + 9x - 13$ — Combine like terms

7. $(3x^2y - 2xy^2 + 9xy) + (2xy^2 - 4x^2y + 3xy)$
8. $(5ab^2 - 2a^2b^2 + 3a) - (4a^2b^2 + 3ab^2)$
9. $(5a^2 - b^2) - (4a^2 + 3b^2) - (a^2 - 7b^2)$
10. $(7x^3 - 2x^2y + 4xy^2 - 6y^3) - (4x^3 - 3x^2y - 2xy^2 - y^3)$
11. $(12x - 24yz) + (46yz - 16x - 26z)$
12. $(19x + 3y) - (-22x - 6y)$
13. $-(5a^2b - 6ab + 16c) + (7ab - 4a^2b)$
14. $(6ab + 11b^2c) - (11ab - 6bc)$
15. $-(14x - 31y) - (14x - 6y + 3z)$
16. $(x^2 - 3x + z) - (x^2 + 5x - 8) + (3x^2 - 4x)$
17. $(2x^2 - 7x + 3) + (5x^2 - 6) - (4 - 3x^2)$
18. $(7x^2 - 2xy + y^2) + (3x^2 - 6y^2 + 3xy) - (4y^2 - 6x^2 - xy)$
19. $(5xy - y^2) - (3yz + 2xy) + (3y^2 - 4xy)$
20. $(5ab + 2b^2) - (4a^2 - 3b^2) + (2ab - 7b^2)$

Perform the following addition and subtraction in column form. See example 1–6 B.

21. $\begin{array}{r} (3x^2 + 4xy - 5y^2) \\ + \ \underline{(\ x^2 - 7xy + 3y^2)} \end{array}$

22. $\begin{array}{r} (4a^2 + 2ab + 3b^2) \\ + \ \underline{(\ a^2 - 5ab + \ b^2)} \end{array}$

23. $\begin{array}{r} (3x^2y - 2x^2y^2 + 3xy^2) \\ - \ \underline{(2x^2y + xy - 5xy^2)} \end{array}$

24. $\begin{array}{r} (4st^2 + 3st - 2s^2t) \\ - \ \underline{(2st^2 + 3st - 2s^2t)} \end{array}$

25. $\begin{array}{r} (-6a^2b^2 + 3ab - 4) \\ - \ \underline{(-4a^2b^2 + 2ab - 7)} \end{array}$

26. $\begin{array}{r} (-\ 9x^2y^2 - 4xy + 13) \\ - \ \underline{(-11x^2y^2 + 5xy - 6)} \end{array}$

Set up the following problems and perform the indicated addition and subtraction.

Example Subtract $5a^2 - 2a + 4$ from $6a^2 - 7a + 3$.

Solution
$(6a^2 - 7a + 3) - (5a^2 - 2a + 4)$ Write the algebraic expression
$= 6a^2 - 7a + 3 - 5a^2 + 2a - 4$ Change the sign of each term in the second parentheses
$= a^2 - 5a - 1$ Combine like terms

27. Subtract $3x^2 - 2x + 1$ from $5x^2 + 3x - 7$.
28. Subtract $2a^2 - 7a + 3$ from $a^2 - 2a + 4$.
29. Subtract $5a^2 - 6a + 7$ from $5a^2 + 2a - 3$.
30. Subtract $-3t^2 + 4t + 5$ from $8t^2 - 11t + 12$.
31. Subtract $-2x^2 + 3x - 7$ from $7x^2 + 9x - 4$.
32. From $6t^2 - 7t + 14$, subtract $8t^2 - 11t + 6$.
33. From $-7y^2 + 3y + 11$, subtract $2y^2 - 13y + 3$.
34. From $6z^2 + 5z - 4$, subtract $-4z^2 + 3z - 9$.
35. From $4a^2 + 7a - 12$, subtract $-8a^2 + 7a - 5$.
36. Subtract $2x^2 - 9x + 4$ from the sum of $6x^2 + 3x$ and $5x^2 - 4x + 2$.
37. Subtract $a^2 - 7a + 11$ from the sum of $3a^2 - 4$ and $-5a^2 + 6a$.
38. Subtract $-4t^2 - 3t + 11$ from the sum of $8t^2 - 6$ and $-5t^2 + 11t + 5$.
39. Subtract $6xy - y^2$ from the sum of $5x^2 + 3xy$ and $2x^2 - 7xy - y^2$.
40. From the sum of $8a^2 + 11$ and $2a^2 - 7a + 6$, subtract $4a^2 - a - 1$.
41. From the sum of $5x^2 - 11x - 7$ and $-2x^2 + 3x - 4$, subtract $3x^2 - 7x + 5$.
42. From the sum of $-3t^2 + 2t - 6$ and $-5t^2 - 4t - 8$, subtract $-8t^2 + 2t + 4$.
43. From the sum of $-t^2 - 7t + 1$ and $-3t^2 + 4t - 8$, subtract $-4t^2 - 6t - 5$.

Remove all grouping symbols and perform the indicated addition or subtraction. See example 1–6 C.

Example $(5x^2 - 3x + 2) - [2x^2 - (x^2 - 7)]$

Solution
$= 5x^2 - 3x + 2 - [2x^2 - x^2 + 7]$ Remove both sets of parentheses
$= 5x^2 - 3x + 2 - [x^2 + 7]$ Combine like terms within brackets
$= 5x^2 - 3x + 2 - x^2 - 7$ Remove brackets
$= 4x^2 - 3x - 5$ Combine like terms

44. $5a - [3a - (2a - 3)]$
45. $a - 4 + [3a - (2a + 1)]$
46. $4x + [3x - (x + y)]$
47. $7x - [4x + 3y + (x - 2y)]$
48. $3x - [x - y - (7x - 3y)]$
49. $-(3x - 2y) - (x + y) - [2x - y + (3x - 4y)]$
50. $a - [b + (2a - 4b)] + [5a - 3b]$
51. $(2x - 7y) - \{3x - [4y - (2x + 5y)]\}$
52. $6x^2 - [5x + 3x^2 - (4x^2 - 7x)]$
53. $a - \{a - [a - (2a - b) + 3a]\}$
54. $-\{5x - 3y + [2x - (5x - 7y)] + 4y\}$
55. $(3x + x^2) - \{4x^2 - 3x + [2x^2 - x - (5x^2 + 4x)]\}$
56. $2x + [x - (x^2 - y)] - [2x - (x^2 + y)]$
57. $5a - [6a - (2b - 3c)] - (4b - 3a)$
58. $5a - \{(ab + 4c) + 8b - [7a - (3b - 5c)] + 2a\}$
59. $5x^2 - [3x - (2x^2 + x)] - \{x - [3x^2 - (2x^2 + 3x) - x]\}$
60. $-[-3a^2 + (4a - 3) + 5a] - \{7a^2 + [4a - (a^2 - 3) + 5a^2]\}$

61. $-\{4x^2 - (5x + 3) - 2x\} + \{3x^2 - [7x^2 + (2x - 4) - 5x]\}$
62. $4x^2y - \{3xy^2 - [7x^2y^2 + 3xy^2 - (11x^2y + 2x^2y^2)] - 5x^2y\}$
63. $(9x^2y^2 - 4xy^2) - \{8x^2y^2 - [2xy^2 - (-3x^2y + 7x^2y^2)]\}$

If $P(x) = 2x^2 - x + 3$, $Q(x) = 5x - 4$, and $R(x) = 3x^2 + 4x - 7$, express each of the following in terms of x and perform the indicated operations. See example 1–5 F.

Example $[P(x) - Q(x)] + R(x)$

Solution

$= [(2x^2 - x + 3) - (5x - 4)] + (3x^2 + 4x - 7)$	Substitute
$= [2x^2 - x + 3 - 5x + 4] + 3x^2 + 4x - 7$	Remove parentheses
$= [2x^2 - 6x + 7] + 3x^2 + 4x - 7$	Combine like terms
$= 2x^2 - 6x + 7 + 3x^2 + 4x - 7$	Remove brackets
$= 5x^2 - 2x$	Combine like terms

64. $P(x) + Q(x) + R(x)$
65. $P(x) - Q(x) + R(x)$
$\boxed{66.}$ $P(x) - [Q(x) + R(x)]$
67. $Q(x) - [P(x) + R(x)]$
68. $R(x) - [Q(x) - P(x)]$
69. $[Q(x) - R(x)] - P(x)$
70. $-R(x) + [Q(x) - P(x)]$
71. $-P(x) + [Q(x) - R(x)]$

Chapter 1 lead-in problem

Theresa is conducting a chemistry experiment involving yellow phosphorus. The lab manual states that yellow phosphorus ignites at 34 degrees Celsius. What will the temperature be in degrees Fahrenheit? The formula for changing the temperature measured in degrees Celsius to degrees Fahrenheit is

$$F = 1.8C + 32$$

Solution

$F = 1.8C + 32$	Original expression
$F = 1.8(\quad) + 32$	Formula ready for substitution
$F = 1.8(34) + 32$	Substitute
$F = 61.2 + 32$	Multiply
$F = 93.2$	Add

The temperature is 93.2 degrees Fahrenheit.

Chapter 1 summary

1. A **set** is any collection of objects or things.
2. The set A is a **subset** of the set B, written $A \subseteq B$, if every element of A is also an element of B.
3. The set A is said to be **equal** to the set B, written $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.
4. A set that contains no elements is called the **empty set** or the **null set** and is denoted by the symbol $\emptyset$.
5. The **union** of the sets A and B, written $A \cup B$, is the set of all elements that are in A or in B or in both A and B.
6. The **intersection** of the sets A and B, written $A \cap B$, is the set of only those elements that are in both A and B.
7. The sets A and B are **disjoint** if and only if $A \cap B = \emptyset$.
8. A **variable** is a symbol (generally a lowercase letter) that acts as a placeholder for an unspecified number.
9. The **absolute value** of a number is the undirected distance that the number is from the origin. In symbols
$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$
10. The numbers or variables in an indicated multiplication are referred to as the **factors** of the product.
11. The **exponent** tells how many times the base is used as a factor in an indicated product.
12. **Division by zero is not allowed.**
13. The quotient of *zero* divided by any number other than zero is always zero.
14. **Order of operations**
    1. **Groups** Perform any operations within a grouping symbol such as ( ) parentheses, [ ] brackets, { } braces, | | absolute value, or in the numerator or the denominator of a fraction.

2. **Exponents** Perform operations indicated by exponents.
3. **Multiplication and division** Perform multiplication and division in order from left to right.
4. **Addition and subtraction** Perform addition and subtraction in order from left to right.

15. An **algebraic expression** is any meaningful collection of variables, constants, grouping symbols, and symbols of operations.

16. A **term** is any constant, variable, or indicated product, quotient, or root of constants and variables.

17. **Like terms** are terms that may differ only in their numerical coefficients.

18. Each factor or group of factors is called the **coefficient** of the remaining factors. In the expression $5x$, 5 is called the **numerical coefficient.**

19. A **polynomial** is an algebraic expression that consists of one or more terms, has real number coefficients, all variables are raised only to natural number exponents, and the operations involving variables are limited to addition, subtraction, and multiplication.

20. The **degree** of a polynomial in one variable is the greatest exponent of that variable in any one term.

Chapter 1 error analysis

1. Sets
Example: Write the letters of the word "Mississippi" as a set {m,i,s,s,i,s,s,i,p,p,i}
Correct answer: {M,i,s,p}
What error was made? *(see page 1)*

2. Intersection of sets
Example: $\{1,2,3,4\} \cap \{3,5,6,7\} = \emptyset$
Correct answer: $\{1,2,3,4\} \cap \{3,5,6,7\} = \{3\}$
What error was made? *(see page 3)*

3. Sets of numbers and set-builder notation
Example: $\{x|x$ is a whole number less than $5\} = \{1,2,3,4\}$
Correct answer: $\{x|x$ is a whole number less than $5\}$ $= \{0,1,2,3,4\}$
What error was made? *(see page 5)*

4. Use of "≈" (is approximately equal to)
Example: $3\frac{3}{4} \approx 3.75$
Correct answer: $3\frac{3}{4} = 3.75$
What error was made? *(see page 6)*

5. Absolute value of a real number
Example: $-|-5| = 5$
Correct answer: $-|-5| = -5$
What error was made? *(see page 10)*

6. Exponential form
Example: $-3^2 = 9$
Correct answer: $-3^2 = -(3 \cdot 3) = -9$
What error was made? *(see page 16)*

7. Division involving zero
Example: $\frac{-5}{0} = 0$
Correct answer: $\frac{-5}{0}$ is undefined while $\frac{0}{-5} = 0$.
What error was made? *(see page 17)*

8. Order of operations
Example: $10 \div 2 - 6 \cdot 2 + 3 = 1$
Correct answer: -4
What error was made? *(see page 28)*

9. Coefficients of variables
Example: The coefficients of the expression $4x^3 - 5x^2 - 3x$ are 4, 5, and 3.
Correct answer: The coefficients are 4, -5, and -3.
What error was made? *(see page 32)*

10. Names of polynomials
Example: The polynomial $x - (2 + \sqrt{3})$ is a trinomial.
Correct answer: $x - (2 + \sqrt{3})$ is a binomial.
What error was made? *(see page 33)*

Chapter 1 critical thinking

If you add any three consecutive even integers, the sum will be a multiple of 3. Why is this true?

Chapter 1 review

[1–1]

Write each set by listing the elements.

1. The letters in the word *college*

2. The first 3 months of the year

3. The months that begin with the letter *L*

Use mathematical symbols to write the following.

4. B is a subset of D.

5. The null set is a subset of A.

6. 5 is an element of C.

7. B is not a subset of C.

Use the following sets to determine if exercises 8–10 are true or false and to form the indicated sets in exercises 11–13.
$A = \{1,2,3,4\}$, $B = \{2,3,4,5,6\}$, $C = \{5,6,7,8\}$, $D = \{3,4\}$.

8. $4 \in A$ **9.** $D \subseteq B$ **10.** $A \subseteq B$ **11.** $A \cup B$ **12.** $A \cap B$ **13.** $A \cap C$

Replace the comma with the proper strict inequality symbol, $<$ or $>$, between the following numbers to get a true statement.

14. 0, 4 **15.** $-3, -5$ **16.** $-3, 0$

[1–2]

Find each sum or difference.

17. $(-5) + (-8)$ **18.** $0 - (-8)$ **19.** $8 - 11$

20. $0 + (-2) - 4 + (-3)$ **21.** $(-4) - (-10)$ **22.** $6 - 8 + 7 - 11 - 4$

Perform the indicated operations if possible.

23. $\dfrac{(-35)}{(-7)}$ **24.** $\dfrac{(-18)(-2)}{(-6)}$ **25.** $\dfrac{(-24)}{(+8)}$ **26.** $\dfrac{(-8)(0)}{(-5)(-4)}$

27. -2^4 **28.** $\dfrac{(-14)(+12)}{(-4)(+7)}$ **29.** $\dfrac{(+30)}{(-6)}$

[1–3]

Identify which property is being used in the problem.

30. $(-6) + 6 = 0$

31. $5(a + b) = 5a + 5b$

32. $\left(\dfrac{3}{4}\right)\left(\dfrac{4}{3}\right) = 1$

33. $(3a)b = 3(ab)$

[1–4]

Perform the indicated operations and simplify.

34. $12 - 3 \cdot 4 - 6 + 8$

35. $10 - 16 \div 4 + 7 - 3^2$

36. $26 \div 13 + 8 - 2 \cdot 7$

37. $8 - (4 - 7)^2 + 5 \cdot 6$

38. $15 - (18 - 11)^2 \cdot 2 + 12$

39. $10 - 2[18 - 2(4 - 6) + 11] - 16$

40. $\dfrac{5(4 - 9)}{3} - \dfrac{(-24)}{8}$

41. $\left[\dfrac{(-12) + (-9)}{(-7)}\right]\left[\dfrac{(-8)(-6)}{12}\right]$

42. $\dfrac{5}{8} + \dfrac{1}{2} \div \dfrac{3}{4}$

43. $(2.6) \cdot (5.4) \div (1.8) + 5.7$

44. $(18.47 + 26.89) \div 5.6 + (1.3)^2$

[1–5]

Specify the number of terms in each expression and determine if the expression is a polynomial.

45. $5x^3 - 2x^2 + 7x - 4$

46. $\dfrac{5x^2 - 2x}{4}$

Evaluate the following expressions if $a = -2$, $b = 5$, $c = 3$, and $d = -4$.

47. $2a^2b - cd^2$ **48.** $(b - 2c)^2$ **49.** $(a - 2b)(3c + d)$ **50.** $(ac)^2 - (bd)^2$

51. If $P(x) = x^2 - x - 1$,
find (a) $P(-2)$, (b) $P(0)$, (c) $P(3)$, (d) $P(2) \cdot P(-3)$.

Evaluate the following formulas using the given values.

52. $A = \frac{1}{2}bh$, $b = 20$ and $h = 7$

53. $A = \frac{1}{2}h(b_1 + b_2)$, $h = 10$, $b_1 = 6$, and $b_2 = 9$

54. $R_t = \frac{R_1R_2}{R_1 + R_2}$, $R_1 = 4$ and $R_2 = 6$

Write an algebraic expression for each of the following.

55. The sum of y and 4

56. 12 less than a

57. 7 times the sum of a and 6

58. x times 5 and that product decreased by 3

59. 5 times the sum of a number and 12

60. A number increased by 4 and that sum divided by 8

[1–6]

Perform the indicated addition or subtraction.

61. $7y^2 - 6y + 4y^2 - 3y + 2y^2$

62. $7x^2y + 2xy^2 - 3xy^2 + 7x^2y + 5x^2y^2$

63. $3x + 5x + 8 + 9$

64. $7a + a - 4a - 7 + 3$

65. $3x - 5x + 7x - 6 + 4$

66. $12c + 8c - 9c + 8 - 14$

Remove all grouping symbols and perform the indicated addition or subtraction.

67. $(3x^2 - 2x + 7) - (x^2 + 3x - 4)$

68. $7a - [2a - (3a + 4)]$

69. $2x^2 - [4x + 5x^2 - (3x^2 - 6x)]$

70. $(2a^2 - b) - \{5a^2 - (a^2 - 2b)\}$

Chapter 1 test

Use the following sets to determine if exercises 1–3 are true or false and to form the indicated sets in exercises 4–6. $A = \{2,4,6\}$, $B = \{4,6,8\}$, and $C = \{1,2,3,4\}$.

1. $A \subseteq C$ **2.** $4 \in B$ **3.** $B \not\subseteq C$ **4.** $A \cup B$ **5.** $A \cap B$ **6.** $A \cap C$

Perform the indicated operations if possible.

7. $-6 - 8$

8. $\frac{-18}{3}$

9. $(-3)(-3)$

10. $(2x - 1) - (3x + 4)$

11. -3^2

12. $(5)(0)(-2)(3)$

13. $3a + 5a - a$

14. $25 - (18 - 3 \cdot 4) - 3^2$

15. $15 \div 3 - 2 \cdot 4$

16. $(-2)^4$

17. $8 - (-4) - 7 + 3$

18. $\frac{3}{4} + \frac{1}{2} \cdot \frac{2}{3} - \frac{2}{3}$

19. $\frac{-4 + 4}{4}$

20. $5a + b - a + 4b$

21. If $P(x) = 2x^2 - 3x + 1$, find $P(-2)$.

Evaluate the following expressions if $a = -3$, $b = 2$, and $c = 4$.

22. $a^2 - b^2$

23. $ab - ac$

Write an algebraic expression for each of the following.

24. A number diminished by 3

25. A number increased by 5 and that sum divided by 8

Answers and Solutions

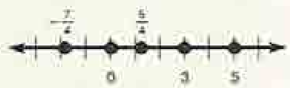
Chapter 1

Exercise 1–1

Answers to odd-numbered problems

1. {Sunday,Saturday} **3.** {10,12,14} **5.** $\emptyset$ **7.** $A \subseteq D$ **9.** $\emptyset \subseteq B$ **11.** {6,7,8} **13.** true **15.** true **17.** false **19.** true **21.** true **23.** {1,2,3,4} **25.** {1,2,3,4,7,8,9} **27.** $\emptyset$ **29.** {2,3,4} **31.** {1,2,3,4}

33. 

35. 

37. 

39. 3 **41.** 0 **43.** -2 **45.** -4 **47.** $<$ **49.** $>$ **51.** $<$ **53.** $<$ **55.** $H \cap Q = \emptyset$ **57.** $\{8\} \subseteq N$ **59.** $\{8\} \notin N$ **61.** yes **63.** no **65.** yes **67.** yes **69.** no **71.** yes **73.** yes **75.** yes

Solutions to trial exercise problems

4. $\emptyset$. There are no months with less than 28 days. **11.** {6,7,8}. The elements are inside the braces.

31. $\{1,2,3,4\} \cup (\{2,3,4\} \cap \{7,8,9\}) = \{1,2,3,4\} \cup \emptyset = \{1,2,3,4\}$

36. 

Using a calculator, we approximate $-\sqrt{3}$ as -1.732 and $\sqrt{2}$ as 1.414. **43.** $-|-2| = -2$. The negative sign in front of the absolute value bar remains. **52.** $0 > -5$. 0 is greater than -5 since 0 is to the right of -5 on the number line. **58.** $8 \not\subseteq N$. We place a slash mark, /, through $\subseteq$ to negate it.

Exercise 1–2

Answers to odd-numbered problems

1. 2 **3.** -14 **5.** 1 **7.** 18 **9.** -7 **11.** -4 **13.** -20 **15.** -12 **17.** 13 **19.** -9 **21.** -17 **23.** 10 **25.** -28 **27.** -27 **29.** -24 **31.** 48 **33.** -240 **35.** 60 **37.** 0 **39.** -64 **41.** 36 **43.** -64 **45.** -8 **47.** 2 **49.** -4 **51.** 3 **53.** -9 **55.** indeterminate **57.** 10 **59.** 0 **61.** 4 **63.** undefined **65.** -4 **67.** $-\$88$ **69.** 3 **71.** 5 yd **73.** $7(-10) = -70$ **75.** $-24°F$ **77.** \$68 **79.** 32 **81.** 240¢ = \$2.40 **83.** 290 ml **85.** 8,708 feet **87.** \$1.80 **89.** 34 miles **91.** 133 over 78 **93.** 27 **95.** 2 mb gain

Solutions to trial exercise problems

9. $(-7) - 0 = -7$. We can subtract zero from any number and the number will be our answer. **14.** $6 - 9 + 11 - 8$
$= 6 + (-9) + 11 + (-8) = (-3) + 11 + (-8) = 8 + (-8)$
$= 0$ **21.** $[(-6) - 5] + 4 - (16 - 6)$
$= [(-6) + (-5)] + 4 - [16 + (-6)] = (-11) + 4 - 10$
$= (-11) + 4 + (-10) = (-7) + (-10) = -17$
36. $(-9)(+2)(0)(-4) = 0$. When zero is one of the factors, the product will be zero. **38.** $-5^2 = -(5 \cdot 5) = -(25) = -25$
59. $\dfrac{(-6)(0)}{(-2)} = \dfrac{0}{(-2)} = 0$ **62.** $\dfrac{(-8)(-6)}{(-2)(0)} = \dfrac{48}{0} =$ undefined

Exercise 1–3

Answers to odd-numbers problems

1. $3y + 4x$ **3.** $4 + (2 + 6)$ **5.** $a \cdot 5$ **7.** $2(xy)$ **9.** 0 **11.** $x + 5$ **13.** $2a - 3$ **15.** 7 **17.** $a \geq 5$ **19.** $a < b$ **21.** $x > 5$ or $x < 5$ **23.** $a > 6$ **25.** closure property for addition **27.** identity property of multiplication **29.** multiplicative inverse property **31.** identity property of addition **33.** associative property of addition **35.** associative property of multiplication **37.** identity property of addition **39.** commutative property of addition **41.** associative property of addition **43.** identity property of addition **45.** $-6, \dfrac{1}{6}$ **47.** $-5, \dfrac{1}{-5}$ **49.** $-x, \dfrac{1}{x}$

51. $7, -7$ **53.** $3, \dfrac{1}{3}$ **55. a.** given **b.** closure property of multiplication **c.** reflexive property of equality **d.** given **e.** substitution property of equality **57. a.** given **b.** additive inverse property **c.** additive inverse property **d.** uniqueness of additive inverse

Solutions to trial exercise problems

22. $4 \geq y$. From trichotomy, if 4 is not less than y, it must be greater than or equal to y. **36.** commutative property of addition because the order in which we added the numbers was changed

Exercise 1–4

Answers to odd-numbered problems

1. 2 **3.** 37 **5.** -8 **7.** 41 **9.** 0 **11.** 10 **13.** 1 **15.** -2 **17.** 25 **19.** 41 **21.** 19 **23.** 9.38 **25.** $-\dfrac{7}{24}$

27. -10.74 **29.** 108.05 **31.** $\dfrac{2}{17}$ **33.** 155 **35.** 45

37. 17 **39.** 5 **41.** 80 **43.** 1 **45.** 100 square meters **47.** 5,313.6 pounds per square inch **49.** 262 square feet **51.** 77 **53.** \$93 **55.** \$176,000 **57.** \$2,547 **59.** 4,900 words

Solutions to trial exercise problems

10. $8 + 0(5 - 7) = 8 + 0(-2) = 8 + 0 = 8$ **20.** $12 + 3 \cdot 16 \div 4^2 - 5 = 12 + 3 \cdot 16 \div 16 - 5 = 12 + 48 \div 16 - 5 = 12 + 3 - 5 = 15 - 5 = 10$ **33.** $5[20 - 3(4 - 6) + 5] = 5[20 - 3(-2) + 5] = 5[20 - (-6) + 5] = 5[26 + 5] = 5[31] = 155$ **41.** $\left[\frac{10 + (-2)}{2(-1)}\right]\left[\frac{(-10)(-4)}{(-2)}\right] = \left[\frac{8}{(-2)}\right]\left[\frac{40}{(-2)}\right] = (-4)(-20) = 80$

Exercise 1–5

Answers to odd-numbered problems

1. 3 terms, polynomial **3.** 3 terms, polynomial **5.** 1 term, not a polynomial since a variable appears under the radical symbol **7.** $4, -7, 1$ **9.** $5, 1, -7$ **11.** trinomial, 3rd degree **13.** binomial, 4th degree **15.** monomial, 5th degree **17.** -12 **19.** 60 **21.** 31 **23.** -7 **25.** -22 **27.** $Q(2) = 2$, $Q(-3) = 22$, $Q(0) = 4$ **29.** $P(1) = 0$, $P(-2) = -9$, $P(0) = -1$ **31.** 6 **33.** 31 **35.** -42 **37.** 4 **39.** -9 **41.** 60 **43.** 55 **45.** 83 **47.** 144 **49.** $\frac{79}{19}$ or $4\frac{3}{19}$ **51.** 12 **53.** 21 **55.** 38 m **57.** $\frac{1{,}280}{3}$ or $426\frac{2}{3}$ rpm **59.** 2.5 amperes **61.** $3x$ **63.** $x + 8$ **65.** $4(x + 7)$ **67.** $x - 4$ **69.** let n = the number; $n - 15$ **71.** let n = the number; $8n + 14$ **73.** let n = the number, $4(n + 3)$ **75.** let n = the number; $\frac{1}{4}n$ **77.** let n = the number; $\frac{1}{2}n$

Solutions to trial exercise problems

4. $\frac{4a^2 - b^2}{10}$ has one term since the fraction bar is a grouping symbol, and it is a polynomial because division by a constant is allowed.
8. 3 is the numerical coefficient of x^3, -2 is the numerical coefficient of x^2, and 1 is understood to be the numerical coefficient of x.
19. $[4(\ \ ) + 3(\ \ )][(\ \ ) - 2(\ \ )] = [4(-3) + 3(2)][(-2) - 2(4)] = [(-12) + 6][(-2) - 8] = [(-12) + 6][(-2) + (-8)] = (-6)(-10) = 60$ **24.** $[3(\ \ ) - 2(\ \ )] - [2(\ \ ) + (\ \ )][(\ \ ) + (\ \ )] = [3(-3) - 2(2)] - [2(-3) + (2)][(-2) + (4)] = [(-9) - (4)] - [(-6) + (2)][2] = [(-9) + (-4)] - [-4][2] = [-13] - [-8] = [-13] + [8] = -5$ **36.** $P[Q(2)] = P[2(2) - 1] = P(4 - 1) = P(3) = (3)^2 + 2(3) + 1 = 9 + 2(3) + 1 = 9 + 6 + 1 = 15 + 1 = 16$
42. $I = (\ \ )(\ \ )(\ \ ) = (2{,}000)(0.09)(3) = 540$
49. $A = \frac{(\ \ )(\ \ ) + (\ \ )(\ \ )}{(\ \ ) + (\ \ )} = \frac{(80)(3) + (110)(5)}{(80) + (110)} = \frac{240 + 550}{190} = \frac{790}{190} = \frac{79}{19}$ or $4\frac{3}{19}$

Exercise 1–6

Answers to odd-numbered problems

1. $-x^2 + 8x$ **3.** $-5a^3 + a^2 + 5a$ **5.** $4x^2y + 3xy$ **7.** $-x^2y + 12xy$ **9.** $3b^2$ **11.** $-4x + 22yz - 26z$ **13.** $-9a^2b + 13ab - 16c$ **15.** $-28x + 37y - 3z$ **17.** $10x^2 - 7x - 7$ **19.** $2y^2 - xy - 3yz$ **21.** $4x^2 - 3xy - 2y^2$ **23.** $x^2y - 2x^2y^2 - xy + 8xy^2$ **25.** $-2a^2b^2 + ab + 3$ **27.** $2x^2 + 5x - 8$ **29.** $8a - 10$ **31.** $9x^2 + 6x + 3$ **33.** $-9y^2 + 16y + 8$ **35.** $12a^2 - 7$ **37.** $-3a^2 + 13a - 15$ **39.** $7x^2 - 10xy$ **41.** $-x - 16$ **43.** $3t - 2$ **45.** $2a - 5$ **47.** $2x - y$ **49.** $-9x + 6y$ **51.** $-3x - 8y$ **53.** $2a + b$ **55.** $11x$ **57.** $2a - 2b - 3c$ **59.** $8x^2 - 7x$ **61.** $-8x^2 + 10x + 7$ **63.** $-6x^2y^2 + 3x^2y - 2xy^2$ **65.** $5x^2 - 2x$ **67.** $-5x^2 + 2x$ **69.** $-5x^2 + 2x$ **71.** $-5x^2 + 2x$

Solutions to trial exercise problems

13. $-(5a^2b - 6ab + 16c) + (7ab - 4a^2b) = -5a^2b + 6ab - 16c + 7ab - 4a^2b = (-5a^2b - 4a^2b) + (6ab + 7ab) - 16c = -9a^2b + 13ab - 16c$

23.
$$\begin{array}{l} (3x^2y - 2x^2y^2 + 3xy^2) \\ -\underline{(2x^2y + \quad xy - 5xy^2)} \end{array} = \begin{array}{rrrr} 3x^2y & -\ 2x^2y^2 & & +\ 3xy^2 \\ -\ 2x^2y & & -\ xy & +\ 5xy^2 \\ \hline x^2y & -\ 2x^2y^2 & -\ xy & +\ 8xy^2 \end{array}$$

27. $(5x^2 + 3x - 7) - (3x^2 - 2x + 1) = 5x^2 + 3x - 7 - 3x^2 + 2x - 1 = 2x^2 + 5x - 8$ **32.** $(6t^2 - 7t + 14) - (8t^2 - 11t + 6) = 6t^2 - 7t + 14 - 8t^2 + 11t - 6 = -2t^2 + 4t + 8$
36. $[(6x^2 + 3x) + (5x^2 - 4x + 2)] - (2x^2 - 9x + 4) = [6x^2 + 3x + 5x^2 - 4x + 2] - (2x^2 - 9x + 4) = (11x^2 - x + 2) - (2x^2 - 9x + 4) = 11x^2 - x + 2 - 2x^2 + 9x - 4 = 9x^2 + 8x - 2$ **40.** $[(8a^2 + 11) + (2a^2 - 7a + 6)] - (4a^2 - a - 1) = [8a^2 + 11 + 2a^2 - 7a + 6] - (4a^2 - a - 1) = [10a^2 - 7a + 17] - (4a^2 - a - 1) = 10a^2 - 7a + 17 - 4a^2 + a + 1 = 6a^2 - 6a + 18$
49. $-(3x - 2y) - (x + y) - [2x - y + (3x - 4y)] = -3x + 2y - x - y - [2x - y + 3x - 4y] = -4x + y - [5x - 5y] = -4x + y - 5x + 5y = -9x + 6y$
54. $-\{5x - 3y + [2x - (5x - 7y)] + 4y\} = -\{5x - 3y + [2x - 5x + 7y] + 4y\} = -\{5x - 3y + [-3x + 7y] + 4y\} = -\{5x - 3y - 3x + 7y + 4y\} = -\{2x + 8y\} = -2x - 8y$
60. $-[-3a^2 + (4a - 3) + 5a] - \{7a^2 + [4a - (a^2 - 3) + 5a^2]\} = -[-3a^2 + 4a - 3 + 5a] - \{7a^2 + [4a - a^2 + 3 + 5a^2]\} = -[-3a^2 + 9a - 3] - \{7a^2 + [4a^2 + 4a + 3]\} = 3a^2 - 9a + 3 - \{7a^2 + 4a^2 + 4a + 3\} = 3a^2 - 9a + 3 - \{11a^2 + 4a + 3\} = 3a^2 - 9a + 3 - 11a^2 - 4a - 3 = -8a^2 - 13a$
66. $(2x^2 - x + 3) - [(5x - 4) + (3x^2 + 4x - 7)] = 2x^2 - x + 3 - [5x - 4 + 3x^2 + 4x - 7] = 2x^2 - x + 3 - [3x^2 + 9x - 11] = 2x^2 - x + 3 - 3x^2 - 9x + 11 = -x^2 - 10x + 14$

Chapter 1 review

1. {c,o,l,e,g} **2.** {January,February,March} **3.** $\emptyset$ **4.** $B \subseteq D$ **5.** $\emptyset \subseteq A$ **6.** $5 \in C$ **7.** $B \not\subseteq C$ **8.** true **9.** true **10.** false **11.** {1,2,3,4,5,6} **12.** {2,3,4} **13.** $\emptyset$ **14.** $<$ **15.** $>$ **16.** $<$ **17.** -13 **18.** 8 **19.** -3 **20.** -9 **21.** 6 **22.** -10 **23.** 5 **24.** -6 **25.** -3 **26.** 0 **27.** -16 **28.** 6 **29.** -5 **30.** additive inverse property **31.** distributive property of multiplication over addition **32.** multiplicative inverse property **33.** associative property of multiplication **34.** 2 **35.** 4 **36.** -4 **37.** 29 **38.** -71 **39.** -72 **40.** $-\frac{16}{3}$ or $-5\frac{1}{3}$ **41.** 12 **42.** $\frac{31}{24}$ or $1\frac{7}{24}$ **43.** 13.5 **44.** 9.79 **45.** 4 terms, polynomial **46.** 1 term, polynomial **47.** -8 **48.** 1 **49.** -60 **50.** -364 **51.** (a) 5, (b) -1, (c) 5, (d) 11 **52.** 70 **53.** 75 **54.** $\frac{12}{5}$ or $2\frac{2}{5}$ **55.** $y + 4$ **56.** $a - 12$ **57.** $7(a + 6)$ **58.** $5x - 3$ **59.** let n = the number; $5(n + 12)$

60. let n = the number; $\frac{n+4}{8}$ **61.** $13y^2 - 9y$
62. $14x^2y + 5x^2y^2 - xy^2$ **63.** $8x + 17$ **64.** $4a - 4$
65. $5x - 2$ **66.** $11c - 6$ **67.** $2x^2 - 5x + 11$ **68.** $8a + 4$
69. $-10x$ **70.** $-2a^2 - 3b$

Chapter 1 test

1. false **2.** true **3.** true **4.** $\{2,4,6,8\}$ **5.** $\{4,6\}$ **6.** $\{2,4\}$
7. -14 **8.** -6 **9.** 9 **10.** $-x - 5$ **11.** -9 **12.** 0
13. $7a$ **14.** 10 **15.** -3 **16.** 16 **17.** 8 **18.** $\frac{5}{12}$ **19.** 0
20. $4a + 5b$ **21.** 15 **22.** 5 **23.** 6 **24.** let x = the number; $x - 3$ **25.** let x = the number; $\frac{x+5}{8}$

Chapter 2

Exercise 2–1

Answers to odd-numbered problems

1. $\{3\}$ **3.** $\left\{\frac{5}{2}\right\}$ **5.** $\{6\}$ **7.** $\{3\}$ **9.** $\{12\}$ **11.** $\left\{\frac{32}{3}\right\}$
13. $\{12\}$ **15.** $\left\{\frac{27}{2}\right\}$ **17.** $\{3\}$ **19.** $\{-1\}$ **21.** $\left\{\frac{5}{3}\right\}$
23. $\left\{\frac{1}{3}\right\}$ **25.** $\left\{\frac{15}{14}\right\}$ **27.** $\left\{\frac{5}{3}\right\}$ **29.** $\emptyset$ **31.** $\left\{-\frac{7}{2}\right\}$
33. $\{8\}$ **35.** $\{24\}$ **37.** $\left\{\frac{9}{2}\right\}$ **39.** $\{2\}$ **41.** $\left\{\frac{31}{3}\right\}$ **43.** $\left\{\frac{-6}{7}\right\}$
45. $\{6\}$ **47.** $\{-8.4\}$ **49.** $\{8\}$ **51.** $\emptyset$ **53.** $\ell = 8$
55. $t = 4$ years **57.** $115c$ **59.** $\frac{18}{h}$ **61.** $50h + 25q + 10d + 5n$
63. $3n, 3n - 8$ **65.** $d + 464 - 5m$ **67.** $w + 1$ **69.** $j + 2$
71. $2d + 500$ **73.** $59c + 115b$ **75.** $20(w + 11)$

Solutions to trial exercise problems

18.
$$\begin{aligned} 4x + 5 &= 5 \\ 4x &= 0 \\ x &= 0 \\ &\{0\} \end{aligned}$$

44.
$$\begin{aligned} 5.6z - 22.15 &= 24.33 \\ 5.6z &= 46.48 \\ z &= 8.3 \\ &\{8.3\} \end{aligned}$$

50.
$$\begin{aligned} 3(2x - 1) &= 6x + 7 \\ 6x - 3 &= 6x + 7 \\ -3 &= 7 \text{ (false)} \\ &\emptyset \end{aligned}$$

Review exercises

1. 264 **2.** 600 **3.** 220 **4.** 38.75 **5.** 256 **6.** 38

Exercise 2–2

Answers to odd-numbered problems

1. $R = 9$ **3.** $P = 3{,}000$ **5.** $n = 20$ **7.** $V_1 = 51$ **9.** $t = \frac{I}{pr}$
11. $m = \frac{E}{c^2}$ **13.** $m = \frac{F}{a}$ **15.** $b = \frac{A}{h}$ **17.** $R = \frac{W}{I^2}$
19. $k = V - gt$ **21.** $g = \frac{V - k}{t}$ **23.** $q = \frac{D - R}{d}$
25. $\ell = \frac{px - m}{p}$ **27.** $W = R + 2bc + b^2$ **29.** $a = \frac{V + br^2}{r^2}$
31. $d = \frac{2S - 2an}{n^2 - n}$ **33.** $g = \frac{2Vt - 2S}{t^2}$ **35.** $d = \frac{\ell - a}{n - 1}$
37. $x = \frac{12 - 3y}{2}$ **39.** $y = \frac{-3x}{7}$ **41.** $x = \frac{5y + 6}{10}$
43. $y = \frac{4x - 3}{a}$ **45.** $y = \frac{ax + 3a + 4b}{b}$ **47.** $v = \frac{2s - gt^2}{2t}$
49. $P_1 = \frac{nP_2 - P - c}{n}$

Solution to trial exercise problem

26.
$$\begin{aligned} R &= W - b(2c + b);\ c \\ R &= W - 2bc - b^2 \\ 2bc + R &= W - b^2 \\ 2bc &= W - b^2 - R \\ c &= \frac{W - b^2 - R}{2b} \end{aligned}$$

Review exercises

1. $3x$ **2.** $6(a + 7)$ **3.** $\frac{y - 2}{4}$ **4.** let n = the number; $5n$
5. let n = the number; $n - 12$ **6.** let n = the number; $\frac{n}{8} - 9$

Exercise 2–3

Answers to odd-numbered problems

1. 40,48 **3.** 6,48 **5.** 21,26 **7.** 18 **9.** 84 **11.** -26
13. 17 **15.** 23,58 **17.** 13 **19.** 26,38 **21.** 47,79 **23.** 5,19
25. 22,23,24 **27.** 14,16,18 **29.** $-23, -21, -19$ **31.** 9,36
33. 24 **35.** 29,40 **37.** 21,22 **39.** 8,10,12 **41.** 11,33,5
43. 8,15 **45.** \$10,000 at 8%; \$5,000 at 6% **47.** \$13,000 at 10%; \$13,000 at 12% **49.** \$5,000 at 10%; \$7,000 at 12%
51. \$12,285.71 at 5%; \$5,714.29 at 9% **53.** \$4,000
55. \$14,000 at 14%; \$12,000 at 10% **57.** \$10,000 at 14%; \$8,000 at 9% **59.** \$19,000 at 12%; \$15,000 at 21%
61. $\ell = 28$ feet, $w = 23$ feet **63.** $\ell = 48$ feet, $w = 16$ feet
65. 14 cm, 7 cm, 17 cm **67.** 40 cm^3 of 10% solution, 80 cm^3 of 4% solution **69.** 3 ounces of 60% gold, 9 ounces of 80% gold **71.** 100 3-grain capsules, 100 2-grain capsules
73. 10 liters of 60% solution, 20 liters of 30% solution

Solutions to trial exercise problems

9. Let n = the number.

a number is divided by 4	increased by	6	is	27
$\frac{n}{4}$	$+$	6	$=$	27

$\frac{n}{4} + 6 = 27$	Original equation
$4\left(\frac{n}{4} + 6\right) = 4 \cdot 27$	Multiply by 4 to clear the fraction
$n + 24 = 108$	Distribute the multiplication in the left member
$n = 84$	Subtract 24

The number is 84.

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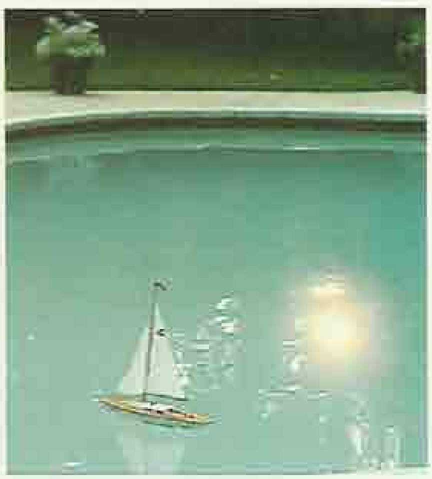

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